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APPLICATION OF PROBABILITY THEORY IN MACHINE SELECTION

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Abstract:
Machine selection plays a crucial role as it impacts various aspects of production, including technical, economic, and environmental factors. Choosing a machine that ensures a balance among all these aspects is a complex task, as each type of machine involves numerous parameters that need consideration, and these parameters may sometimes conflict with one another. This study applies the probability method to select the optimal machine from the available options. It can be asserted that this is the first study utilizing probability for machine selection. The application of probability to choose the optimal machine has been carried out in five cases to select five different types of machines across various fields. The results of machine selection using the probability method have been compared with those obtained using other methods. This study has demonstrated that using probability ensures accuracy in machine selection.

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1. INTRODUCTION

Machinery plays a pivotal role in industrial processing and production. Machines have become indispensable tools in various fields, enhancing labor productivity, saving time, and reducing human labor. They not only increase accuracy in production processes but also ensure uniform product quality [1-2]. Moreover, machinery helps businesses save production costs, boost competitiveness, and expand their scale. With the assistance of machines, production processes have become more efficient, enabling companies to quickly fulfil large and complex orders. In general, machinery is an essential tool in the development and modernization of industry [3]. The rapid development of machine technology has led to the emergence of many advanced and modern machines. These machines are equipped with intelligent and highly automated features, optimizing production processes and minimizing errors. Specifically, machines significantly contribute to mass production in the manufacturing and processing industries, meeting the growing market demands [4, 5]. These factors highlight the critical importance of machine selection, which must be carried out scientifically.

Selecting a machine requires considering multiple parameters simultaneously for each type of machine. However, the parameters for each machine type often differ, sometimes even conflicting. For example, a machine with high power and accuracy tends to be expensive. Focusing solely on either cost or technical specifications may lead to poor machine selection. Instead, machine selection must consider all parameters concurrently, making it a multi-criteria decision-making (*MCDM*) process [6-9]. The application of *MCDM* methods for machine selection has been conducted in numerous studies over the past few decades [10-12], and it has continued to grow recently. A recent review indicated a rapid increase in studies applying *MCDM* methods for machine selection [13]. In particular, for the logistics sector, the application of *MCDM* methods for selecting logistics handling equipment has been strongly encouraged as an alternative to other methods [14].

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However, some studies have indicated that all *MCDM* methods employ "additive" algorithms, and some even contain artificial or subjective elements, rendering these methods semi-quantitative and leading to uncertainty in final decisions [15]. The application of probability for multi-criteria decision-making is a quantitative method, ensuring accuracy and reliability in final decisions and overcoming the aforementioned limitations of *MCDM* methods [15, 16]. Zheng et al. [15] were the first researchers to apply probability to multi-criteria decision-making in 2021, selecting wall construction solutions, project managers, and contractors for construction projects. They concluded that using probability is simpler than using other *MCDM* methods while still ensuring accuracy for the final decision. However, despite significant efforts, it seems that this probabilistic method has not gained much attention from researchers in the application of selecting the best option among available alternatives. Demonstrating the success of probability theory in machine selection is the primary contribution of this research.

2. LITERATURE REVIEW

As presented in Section 1, the application of MCDM methods for machine selection has experienced rapid growth in recent years. Some studies have applied a single *MCDM* method for machine selection, such as using the *FUCA* method to select lathes [17], the *MOORA* method for forklift selection [18], the *TOPSIS* method for choosing wheel loaders [19], the *TOPSIS* method for selecting stackers in the textile industry [20], and the *EDAS* method was utilized for industrial robot selection [21], etc.

However, several studies have noted that the best machine identified using different *MCDM* methods may vary, suggesting that to ensure decision accuracy, machine selection should be conducted using at least two different methods [22]. Following this approach, hydraulic excavator selection was performed using both fuzzy *TOPSIS* and fuzzy *VIKOR* methods [22], a combination of *TOPSIS* and *COPRAS* was used for selecting mixers in a bakery production company [23], flotation machines were chosen using both *TOPSIS* and *VIKOR* methods [24], *MARCOS* and *COCOSO* methods were used for selecting band saws [25], the combination of *DELPHI* and *PROMETHEE* methods was applied for selecting presses in a dairy production facility in Turkey [26], *MOORA* and *MOOSRA* methods were used for ultrasonic machining equipment selection [27], the combination of *COPRAS* and *ARAS* methods was used for selecting industrial robots [28], both the *SAW* and *VIKOR* methods were concurrently employed for spring manufacturing equipment selection [29], and the selection of material handling equipment was conducted by simultaneously applying both the *COPRAS* and ARAS methods [30], etc. Some studies have even combined multiple methods to solve a single machine selection problem. For instance, the *TOPSIS*, *EDAS*, *COCOSO,* and *TODIM* methods were recently integrated to select car wash machines for gas stations [31]. Similarly, the selection of plastic injection molding machines was conducted by simultaneously applying the *PIV*, *PSI*, *FUCA*, and *CURLI* methods [32]. Additionally, robot selection was performed by hybridizing the *TOPSIS*, *COPRAS*, and *ARAS* methods [33], etc.

As demonstrated, *MCDM* methods have been widely applied for machine selection across various domains. However, despite its notable advantages over other *MCDM* methods, as discussed in the introduction, the probabilistic method has not garnered significant attention from researchers in selecting the optimal alternative among available options. This study aims to evaluate the effectiveness of the probabilistic method in machine selection.

Chapter 2 of this paper introduces the application of the probability method in multi-criteria decision-making. This chapter also introduces Spearman's rank correlation coefficient to compare the ranking similarity of alternatives when ranked using the probability method and other *MCDM* methods. The performance evaluation of the probability method in machine selection is conducted in several examples in Chapter 3. The conclusions drawn and future research directions are presented in the final section of this paper.

3. MATERIALS AND METHODS

To find the optimal solution among the available alternatives, the application of the probability method is carried out in the following sequence [15].

Step 1: Establish a summary table regarding the number of alternatives to be ranked as well as the criteria for evaluating each alternative. Let *m* be the number of alternatives to be ranked, *n* be the number of criteria for each alternative, and X_{ij}

be the value of criterion *j* for alternative *i*, where *i = 1 to m*, and *j = 1 to n*. Denote q_j as the weight of criterion *j*, $\sum_{j=1}^{n} q_j = 1$.

Step 2: Compute the probability of a favorable result.

For profit criteria, the probability of achieving favorable results in the decision-making process increases linearly and is calculated according to (1).

$$P_{ij} \infty X_{ij}, \quad P_{ij} = \alpha_j X_{ij}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots n \tag{1}$$

Where α_j is the normalization coefficient of the *j-th* profit criterion, and is calculated according to (2).

$$\alpha_j = \frac{1}{\sum_{i=1}^{m} X_{ij}} \tag{2}$$

For cost criteria, the probability of achieving favorable results in the decision-making process is also a linear function and is calculated according to (3).

$$P_{ij} \infty \left(X_{jmax} + X_{jmin} - X_{ij}\right), \quad P_{ij} = \beta_j \left(X_{jmax} + X_{jmin} - X_{ij}\right), \quad i = 1, 2, \dots, m, j = 1, 2, \dots n \tag{3}$$

Where β_j is the normalization coefficient of the *j-th* cost criterion, and is calculated according to (4).

$$\beta_j = \frac{1}{m\left(X_{jmax} + X_{jmin} - \frac{\sum_{i=1}^{m} X_{ij}}{m}\right)} \tag{4}$$

Step 3: Determine the overall probability of success for each solution using equation (5).

Considering the weight q_j of criterion *j*, the overall favorable probability of alternative *i* is calculated according to (5). The best alternative is the one with the highest favorable probability.

$$P_i = \prod_{j=1}^{n} \left(P_{ij}\right)^{q_j} \tag{5}$$

Step 4: Rank the alternatives based on the principle that the best alternative is the one with the highest overall probability.

The above mathematical expressions of the probability method will be used to rank the alternatives to select the best one among the available options. This task will be carried out in the next chapter for machine selection in various cases. To evaluate the use of the probability method in machine selection, it is necessary to compare the ranking similarity of alternatives when ranked using the probability method versus other *MCDM* methods. In this study, Spearman's rank correlation coefficient will be used to perform this task [34]. This coefficient is calculated according to (6).

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2 - 1)} \tag{6}$$

In (6), D_i represents the difference in the ranking of alternative *i* when ranked by different methods.

4. APPLICATION CASES

To verify whether the application of the probability method is suitable for machine selection, this chapter examines five different cases.

Case 1 involves selecting one option among six types of wood planers (*WP*), each specified by three profit criteria and three cost criteria.

Case 2 involves selecting one option among six types of forklifts (*FOR*), each option including four profit criteria and two cost criteria.

Case 3 involves selecting a welding robot (*WR*) among seven different options, each comprising four profit criteria and two cost criteria.

Case 4 involves selecting a milling machine (*MM*) among nine options, each option having one cost criterion and twelve profit criteria.

Case 5 involves selecting a grinding machine (*GM*) among twelve different options, each option having seven profit criteria and one cost criterion.

It can be seen that these five cases involve selecting machines for different applications. The number of options, criteria, and types of criteria vary in each case. Additionally, to ensure an objective evaluation of the probabilistic method's performance, the datasets used in this study have previously appeared in publications where machine selection was carried out using distinct methodologies. It is noteworthy that the comparative methods vary across different cases. By intentionally selecting these diverse scenarios, we aim to provide a comprehensive assessment of the probabilistic method's efficacy in machine selection. Details of each case are presented below.

Case 1

Table 1 summarizes the data for six types of wood planers to be ranked, designated as *WP1, WP2, WP3, WP4, WP5*, and *WP6*. The six criteria for each alternative include planing width (C1), maximum planing depth (C2), maximum no-load speed (C3), total machine length (C4), weight (C5), and price (C6). The units of each criterion are also placed in the second row of this table. Please note that the unit of criterion C6, "VND", is the Vietnamese currency unit, where one US dollar is approximately equivalent to 24,000 VND. The first three criteria are profit criteria, while the remaining three are cost criteria. The weights of these criteria have been calculated using the *SPC* method and are summarized in the last row of Table 1. Previously, the ranking of these six types of *WPs* was performed using the *CRADIS* and *CURLI* methods [35].

Table 1. Types of WP [35]

Alt.	C1	C2	C3	C4	C5	C6
Unit	mm	mm	m/min	mm	kg	VND
WP1	82	2	16000	285	3	1.586
WP2	82	2.6	16500	300	2.8	1.529
WP3	82	1.8	16000	290	2.5	1.390
WP4	102	1	17000	280	2.7	2.430
WP5	82	2	11500	280	2.7	1.135
WP6	82	3	18000	390	4.6	2.218
Weight	0.0956	0.2063	0.1132	0.1328	0.2351	0.2170

The application of the probability method to rank the *WPs* is performed as follows:

The normalized coefficients α_1, α_2, and α_3 for criteria C1, C2, and C3 have been calculated using Equation (2).

$$\alpha_1 = \frac{1}{\sum_{i=1}^{6} X_{i1}} = 0.001953$$

$$\alpha_2 = \frac{1}{\sum_{i=1}^{6} X_{i2}} = 0.080645$$

$$\alpha_3 = \frac{1}{\sum_{i=1}^{6} X_{i3}} = 0.00011$$

The normalized coefficients β_4, β_5, and β_6 for criteria C4, C5, and C6 have been calculated using Equation (4).

$$\beta_4 = \frac{1}{6\left(X_{4max} + X_{4min} - \frac{\sum_{i=1}^{6} X_{i4}}{6}\right)} = 0.000456$$

$$\beta_5 = \frac{1}{6\left(X_{5max} + X_{5min} - \frac{\sum_{i=1}^{6} X_{i5}}{6}\right)} = 0.041152$$

$$\beta_6 = \frac{1}{6\left(X_{6max} + X_{6min} - \frac{\sum_{i=1}^{6} X_{i6}}{6}\right)} = 0.090074$$

The values P_{ij} of criteria C1, C2, and C3 have been calculated using Eq. (1), for example, the calculation of P_{11} is performed as follows.

$$P_{11} = \alpha_1 X_{11} = 0.001953 \times 82 = 0.16016$$

The values P_{ij} of criteria C4, C5, and C6 have been calculated using Eq. (3), for example, the calculation of P_{14} is performed as follows.

$$P_{14} = \beta_4 (X_{4max} + X_{4min} - X_{14}) = 0.17540$$

The calculation of P_{ij} values for all criteria in all alternatives has been performed similarly, and the results are summarized in Table 2.

Table 2. Probability values P_{ij} for Case 1

Alt.	C1	C2	C3	C4	C5	C6
A1	0.16016	0.16129	0.16842	0.17540	0.16872	0.17826
A2	0.16016	0.20968	0.17368	0.16856	0.17695	0.18339
A3	0.16016	0.14516	0.16842	0.17312	0.18930	0.19591
A4	0.19922	0.08065	0.17895	0.17768	0.18107	0.10223
A5	0.16016	0.16129	0.12105	0.17768	0.18107	0.21888
A6	0.16016	0.24194	0.18947	0.12756	0.10288	0.12133

The overall probability of success P_i for each alternative has been calculated using Eq. (5). For example, the calculation of P_1 for *WP1* is as follows:

$$P_1 = \prod_{j=1}^{6} (P_{1j})^{q_j} = 0.1692$$

All remaining P_i values were calculated in a similar manner and have been summarized in Table 3. The ranking of each alternative has been determined based on their overall probability of success and has been filled in the last column of this table.

Table 3. Overall Favorable Probability and Rank of the WPs

Alt.	$(P_{ij})^{qj}$						Pi	Rank
	C1	C2	C3	C4	C5	C6		
WP1	0.8394	0.6863	0.8174	0.7937	0.6581	0.6878	0.1692	4
WP2	0.8394	0.7245	0.8202	0.7895	0.6655	0.6921	0.1814	1
WP3	0.8394	0.6716	0.8174	0.7923	0.6761	0.7021	0.1733	3
WP4	0.8571	0.5949	0.8230	0.7950	0.6691	0.6096	0.1361	6
WP5	0.8394	0.6863	0.7874	0.7950	0.6691	0.7192	0.1735	2
WP6	0.8394	0.7462	0.8283	0.7608	0.5858	0.6327	0.1463	5

Thus, the probability method was used to rank the six types of *WPs*. Fig. 1 illustrates the rankings of the *WPs* when ranked using the probability method in this study and using the *CRADIS* and *CURLI* methods in [35]. The Spearman correlation coefficient between the two methods was calculated using Eq. (6), and the results are presented in Table 4.

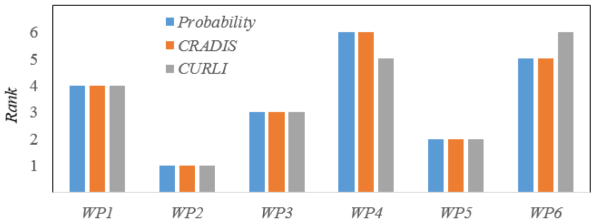

Fig. 1. Ranking of the WPs

Table 4. Spearman coefficient for case 1

Method	Probability	CRADIS	CURLI
Probability	1	1	0.9429
CRADIS		1	0.9429
CURLI			1

It is observed that the rankings of the *WPs* are consistent when ranked using the Probability method and the *CRADIS* method. The Spearman coefficient between Probability and *CRADIS* is 1, indicating no difference in the rankings of the *WPs* when ranked using these two methods. This consistency is also evident in Fig. 1. The machines ranked 1st (*WP2*), 2nd (*WP5*), 3rd (*WP3*), and 4th (*WP1*) are consistent when ranked using all three methods: Probability, *CRADIS*, and *CURLI*. The Spearman coefficient between Probability and *CURLI* is 0.9429, showing a very high similarity in the rankings of the *WPs* when ranked using these two methods. All these findings indicate that the Probability method ensures high accuracy in ranking the *WPs* in this case. *WP2* is conclusively the best type.

Case 2

Table 5 summarizes the data for six types of forklifts (*FOR*). Each alternative is characterized by six criteria: maximum lifting capacity (C1), maximum lifting height (C2), minimum lifting height (C3), fork length (C4), fork width (C5), and price (C6). In the second row of this table, the unit of each criterion is also presented. C3 and C6 are cost criteria, while the remaining four criteria are profit criteria. The weights of the criteria, determined using the entropy method [36], are presented in the final row of this table.

Table 5. Types of FOR [36]

Alt.	C1	C2	C3	C4	C5	C6
Unit	kg	mm	mm	mm	mm	VND
FOR1	2000	200	80	1150	550	4.75
FOR2	2000	200	80	1220	685	4.95
FOR3	2500	200	80	1150	550	4.95
FOR4	2500	200	80	1220	685	5.15
FOR5	3000	200	80	1150	550	5.35
FOR6	3000	220	60	1220	685	5.5
Weight	0.1520	0.1561	0.1616	0.1525	0.1533	0.2245

The ranking of the *FORs* using the Probability method was carried out similarly to Case 1. Fig. 2 illustrates the rankings of the *FORs* when ranked using the Probability method and using the *COCOSO* and *PIV* methods [36]. The Spearman coefficients between the methods were also calculated and summarized in Table 6.

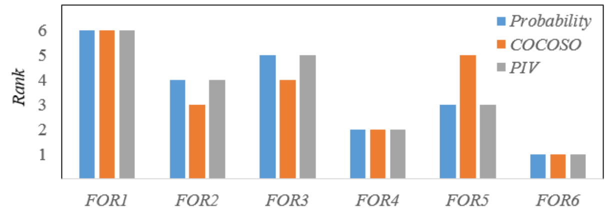

Fig. 2. Ranking of the FORs

Table 6. Spearman coefficient for case 2

Method	Probability	COCOSO	PIV
Probability	1	0.8286	1
COCOSO		1	0.8286
PIV			1

The rankings of the *FORs* using the Probability method are identical to those obtained using the *PIV* method. This is also reflected by the Spearman coefficient of 1 between Probability and *PIV*. Additionally, all three methods, Probability, *COCOSO*, and *PIV,* indicated that *FOR6* ranked 1st, *FOR4* ranked 2nd, and *FOR1* ranked 6th. The Spearman coefficient between Probability and *COCOSO* is also very high (0.8286), indicating that the differences in the rankings of the *FORs* are minimal when comparing the Probability method to the *COCOSO* method. All these findings show that the Probability method ensures high accuracy in ranking the *FORs* in this case. *FOR6* is conclusively the best type.

Case 3

Table 7 summarizes the information for seven types of welding robots that need to be ranked, designated as *WR1* to *WR7*. Each alternative has four profit criteria: horizontal reach (C1), vertical reach (C2), payload capacity (C5), and the number of poles (C6), and two cost criteria: price (C3) and error rate (C4). The units of the criteria have also been listed in the second row of this table. The *MEREC* method was used to calculate the weights of the criteria, and the values of these weights have also been summarized in the last row of this table [37].

Table 7. Types of WR [37]

Alt.	C1	C2	C3	C4	C5	C6
Unit	mm	mm	USD	%	kg	-
WR1	727	1312	6809	4	8	6
WR2	927	1693	6170	6	7	6
WR3	1440	2511	4213	12	12	6
WR4	3121	5616	5319	16	6	6
WR5	2010	3649	6000	16	12	6
WR6	1730	3089	5532	12	25	6
WR7	1434	2475	3553	16	3	7
Weight	0.2594	0.2554	0.0931	0.1775	0.2067	0.0079

Fig. 3 illustrates the rankings of the *WRs* when ranked using the Probability method and the three methods *TOPSIS, RAM*, and *AROMAN* from [37]. The Spearman coefficients between the methods were also calculated and summarized in Table 8.

The Spearman coefficients between the Probability method and the *TOPSIS, RAM*, and *AROMAN* methods are 0.9643, 0.8929, and 0.8571, respectively. All these values are very close to 1, indicating that the rankings of the *WRs* using the Probability method have minimal differences compared to the other methods. Notably, all four methods indicated that *WR6* ranked 1st and *WR3* ranked 4th. The three methods, Probability, *TOPSIS*, and *RAM,* all ranked *WR4* as 2nd and *WR5* as 3rd. These results strongly support the conclusion that the Probability method ensures high accuracy in ranking the *WRs* in this case, with *WR6* being definitively identified as the best type.

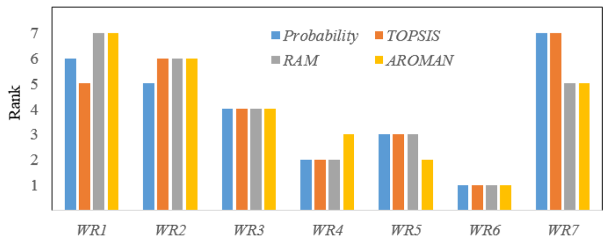

Fig. 3. Ranking of the WRs

Table 8. Spearman coefficients for case 3

Method	Probability	TOPSIS	RAM	AROMAN
Probability	1	0.9643	0.8929	0.8571
TOPSIS		1	0.8571	0.8214
RAM			1	0.9643
AROMAN				1

Case 4

Table 9 summarizes the data of nine milling machines to be ranked, denoted from *MM1* to *MM9*. Thirteen criteria have been used to describe each machine, including [38]:

C1 is the largest dimension of the workpiece's surface that the machine can mill when using a face mill;

C2 is the largest dimension of the workpiece's groove that the machine can mill when using a finger mill;

C3 is the largest diameter of the hole that can be machined when the machine is operated in drilling mode;

C4 is the maximum travel of the machine's spindle. This parameter determines the maximum drilling depth of the machine;

C5 is the number of speed stages, which determines the flexibility in changing cutting speed;

C6 is the motor power;

C7 is the size of the machine table. A larger table allows for processing larger workpieces;

C8 is the vertical travel of the machine table. This parameter determines the maximum length of the machined surface that can be milled in one pass;

C9 is the horizontal travel of the machine table. This parameter determines the maximum width of the machined surface that can be milled in one pass;

C10 is the distance from the spindle to the table. This parameter determines the maximum height of the workpiece that can be clamped to the machine table;

C11 is the distance from the spindle to the column. This parameter determines the maximum width of the workpiece that can be clamped to the machine table;

C12 is the weight of the machine. A heavier machine has higher rigidity;

C13 is the cost of the machine.

The values of the criteria for each alternative are summarized in Table 9. The second row of this table also shows the units of the criteria. The unit of C13 is "million" which is the Vietnamese monetary unit.

Table 9. Types of MM [38]

Alt.	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
Unit	mm	mm	mm	mm	-	W	mm^2	mm	mm	mm	mm	kg	million
MM1	40	10	16	80	12	550	63840	320	120	360	182	100	12.5
MM2	63	13	25	110	5	750	108225	240	140	510	240	190	25
MM3	80	22	31.5	130	12	1500	153300	450	170	460	202	285	40
MM4	80	22	40	110	6	1100	153300	450	170	450	261.5	340	63
MM5	80	22	40	110	12	1100	153300	450	170	450	261.5	340	65
MM6	80	28	45	120	6	1110	196800	540	170	475	260	318	57
MM7	80	28	32	110	6	1500	153300	450	170	475	260	405	68
MM8	80	28	32	110	12	1500	196800	540	170	475	260	405	70
MM9	80	28	45	110	6	1500	196800	540	170	475	260	375	82

The ranking results of the *MMs* using the Probability method and the *FUCA* and *CURLI* methods in [38] are illustrated in Fig 4.

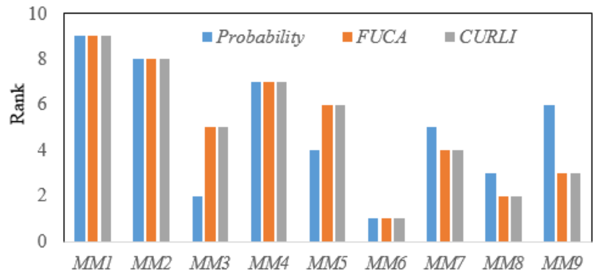

Fig. 4. Ranking of the *MMs*

The Spearman correlation coefficient between the Probability method and the two methods *FUCA* and *CURLI* is 0.8. This result also shows that the rankings of the *MMs* when ranked by the three methods Probability, *FUCA*, and *CURLI* have a very high level of similarity. Notably, all three methods indicate that *MM6* ranks 1st, *MM4* ranks 7th, *MM2* ranks 8th, and *MM1* ranks 9th. This result also confirms the accuracy of the Probability method in ranking the *MMs*, and *MM6* is confirmed as the best alternative.

Case 5

The information about the 12 grinding machines (*GM*) that need to be ranked is summarized in Table 10. The eight criteria from C1 to C8 in Table 10 include the maximum travel of the machine table along the X-axis, the maximum travel of the machine table along the Y-axis, the maximum travel of the machine table along the Z-axis, the maximum diameter of the grinding wheel that can be mounted on the machine, the maximum speed of the grinding wheel, the machine's power, the machine's accuracy, and the year of manufacture. Only C7 is a cost criterion, while the other criteria are profit criteria. In the last row of this table, the weight values for each criterion, previously calculated using the equal method [38], have also been added. The unit of each criterion is presented in row 2.

The rankings of the *GMs* using the Probability method in this study and the *FUCA* and *CURLI* methods from [38] are illustrated in Fig. 5. The Spearman coefficients between the methods are summarized in Table 11.

Table 10. Types of GM [38]

Alt.	C1	C2	C3	C4	C5	C6	C7	C8
Unit	mm	mm	mm	mm	m/s	kW	mm	Year
GM1	315	110	300	205	38.5	2.2	0.005	2016
GM2	600	300	350	305	28	3.7	0.005	2016
GM3	600	300	350	305	28	3.7	0.005	1998
GM4	600	400	380	305	28	3.7	0.005	1992
GM5	315	110	300	205	38.5	2.2	0.005	2002
GM6	315	110	300	205	38.5	2.2	0.002	2009
GM7	500	200	350	205	31.5	3.7	0.005	2009
GM8	510	205	355	205	31.5	3.7	0.005	2014
GM9	1280	550	600	510	53.5	3.4	0.002	2017
GM10	600	500	400	355	37	3.7	0.002	2018
GM11	1600	720	650	510	53.5	4.2	0.002	2014
GM12	510	205	355	205	31.5	3.7	0.005	2016
Weight	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250

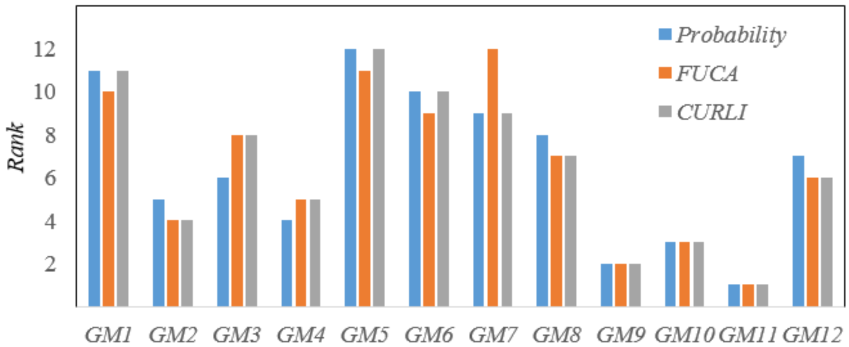

Fig. 5. Ranking of the GM

Table 11. Spearman coefficients for case 5

Method	Probability	FUCA	CURLI
Probability	1	0.9301	0.9720
FUCA		1	0.9580
CURLI			1

The Spearman coefficient between the Probability method and the *FUCA* method is 0.9301, and between the Probability method and the *CURLI* method is 0.9720, indicating a very high similarity in the rankings of the *GMs* when ranked using different methods. Notably, all three methods indicated that *GM11* ranked 1st, *GM9* ranked 2nd, and *GM10* ranked 3rd. This result confirms that the Probability method ensures accuracy in ranking the *GMs* in this case, and *GM11* is definitively identified as the best option.

In the five cases above involving the use of the Probability method for machine selection across various industries, from *WP* for woodworking, *FOR* for warehouse handling operations, *WR* for welding structures, and *MM* and *GM* for machining processes, the Probability method consistently identified the best machines similarly to other *MCDM* methods. Furthermore, calculating the Spearman rank correlation coefficient shows minimal differences in machine rankings when using the Probability method compared to other methods. It is worth reiterating that in each case, the performance of the probabilistic method has been compared with various *MCDM* methods. In specific terms, case 1 compared the Probability method with *CRADIS* and *CURLI* methods, case 2 compared the Probability method with *COCOSO* and *PIV* methods, case 3 compared the Probability method with *TOPSIS*, *RAM*, and *AROMAN* methods, while cases 4 and 5 both compared the Probability method with *FUCA* and *CURLI* methods. These methods have significant algorithmic differences, one of which is the normalization method used for the available data. This is considered a major factor that can lead to differences in the ranking of alternatives when ranked using different *MCDM* methods. *CRADIS* uses linear normalization [39], *COCOSO* uses weitendorf normalization [40], *PIV* and *TOPSIS* [41] use vector normalization, *RAM* uses sum linear normalization [42], *AROMAN* uses both linear and vector normalization [43], while *CURLI* and *FUCA* do not use any normalization [44]. Thus, although the probabilistic method has been compared with many different *MCDM* methods, these methods have different data normalization methods. However, in all comparisons, the probabilistic method has always shown comparable performance to these *MCDM* methods. All these findings demonstrate that the application of the Probability method for optimal machine selection is accurate. Moreover, the Probability method is also expected to succeed in solving multi-objective problems in other fields of research.

5. CONCLUSION

Beyond the technical aspects, machine selection is influenced by a myriad of factors, including productivity, cost, and environmental sustainability. The nuanced characteristics of different machines necessitate a comprehensive evaluation. This research pioneers the use of probabilistic methods in machine selection across various sectors, departing from traditional *MCDM* approaches. The following conclusions were drawn:

- ✓ As a quantitative method, it avoids the use of 'additive' algorithms and does not contain any artificial or subjective elements. This makes the probabilistic approach highly accurate and reliable for final decision-making. Results have shown that applying the Probability method to machine selection guarantees accuracy. This research has proven that applying the probability method to machine selection ensures complete accuracy. The probability method is also believed to be successful in selecting the optimal solution when applied in other fields.
- ✓ For any specific profit criterion, if the sum of all its values across all ranked alternatives is zero, Equation (2) becomes undefined, rendering the probability method inapplicable. This limitation necessitates the development of alternative approaches to expand the method's scope.
- ✓ Furthermore, future research should explore the sensitivity analysis of the probabilistic method when varying weighting schemes for criteria or when the number of ranked alternatives increases or decreases. Additionally, applications of the probabilistic method to machine selection and other domains should be investigated, particularly in cases where the decision matrix contains fuzzy sets or qualitative parameters. Hybrid approaches combining the probabilistic method with other techniques could be explored to leverage each other's strengths.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

REFERENCES

[1] Z. Kang, C. Catal, B. Tekinerdogan, Machine learning applications in production lines: A systematic literature review. *Computers & Industrial Engineering*, 149, 2020: 106773. https://doi.org/10.1016/j.cie.2020.106773

[2] M. Soori, B. Arezoo, R. Dastres, Digital twin for smart manufacturing, A review. *Sustainable Manufacturing and Service Economics*, 2, 2023: 100017. https://doi.org/10.1016/j.smse.2023.100017

[3] M. Gorska, M. Daron, Importance of Machine Modernization in Energy Efficiency Management of Manufacturing Companies. *Energies,* 14(24), 2021: 8383. https://doi.org/10.3390/en14248383

[4] S. Phuyal, D. Bista, R. Bista, Challenges, Opportunities and Future Directions of Smart Manufacturing: A State of Art Review. *Sustainable Futures*, 2, 2020: 100023. https://doi.org/10.1016/j.sftr.2020.100023

[5] M. Soori, R. Dastres, B. Arezoo, F.K.G. Jough, Intelligent robotic systems in Industry 4.0: A review. *Journal of Advanced Manufacturing Science and Technology*, 4(3), 2024: 2024007. https://doi.org/10.51393/j.jamst.2024007

[6] A.B. Moradikhou, M. Ravanshadnia, Evaluation of Wheel Loader Selection Using an Integrated Approach with AHP and TOPSIS. *Advance Researches in Civil Engineering*, 3(4), 2021: 45-55.

[7] D. Tesic, D. Bozanic, A. Puska, A. Milic, D. Marinkovic, Development of the MCDM fuzzy LMAW-Grey MARCOS model for selection of a dump truck. *Reports in Mechanical Engineering*, 4(1) 2023: 1-17. https://doi.org/10.31181/rme20008012023t

[8] H. Li, W. Wang, L.Fan, Q. Li, X. Chen, A novel hybrid MCDM model for machine tool selection using fuzzy DEMATEL, entropy weighting and later defuzzification VIKOR. *Applied Soft Computing Journal*, 91, 2020: 106207. https://doi.org/10.1016/j.asoc.2020.106207

[9] P. Bari, P. Karande, Ranking of sequencing rules in a job shop scheduling problem with preference selection index approach. *Journal of Decision Analytics and Intelligent Computing*, 2(1), 2022: 12-25. https://doi.org/10.31181/jdaic10028042022b

[10] J.-P. Tsai, H.-Y. Cheng, S.-Y. Wang, Y.-C. Kao, Multi-Criteria Decision Making Method for Selection of Machine Tool. *2010 International Symposium on Computer, Communication, Control and Automation (3CA),* Tainan, Taiwan, 2010: 49-52. https://doi.org/10.1109/3CA.2010.5533376

[11] R. Karim, C. L Karmaker, Machine Selection by AHP and TOPSIS Methods. *American Journal of Industrial Engineering*, 4(1), 2016: 7-13. https://doi.org/10.12691/ajie-4-1-2

[12] S. Onut, S. S. Kara, T. Efendıgıl, A hybrid fuzzy MCDM approach to machine tool selection. *Journal of Intelligent Manufacturing*, 19, 2008: 443–453. https://doi.org/10.1007/s10845-008-0095-3

[13] A.M. Hagag, L. S. Yousef, T. F. Abdelmaguid, Multi-Criteria Decision-Making for Machine Selection in Manufacturing and Construction: Recent Trends. *Mathematics 11*(631), 2023. https://doi.org/10.3390/math11030631

[14] A. Jusufbašić, MCDM Methods for Selection of Handling Equipment in Logistics: A Brief Review. *Spectrum of Engineering and Management Sciences*, 1(1), 2023: 13-24. https://doi.org/10.31181/sems1120232j

[15] M. Zheng, H. Teng, Y. Wang, A simple approach for multi-criteria decision-making on basis of probability theory. *Engineering Structures and Technologies*, 13(1), 2021: 26-30. https://doi.org/10.3846/est.2021.18404

[16] M. Zheng, J. Yu, H. Teng, Y. Cui, Y. Wang, Probability-Based Multi-objective Optimization for Material Selection - Second Edition. *Springer*, 2023. https://doi.org/10.1007/978-981-99-3939-8

[17] D.T. Do, X T. Nguyen, X.T. Hoang, Combined piprecia method and modified FUCA method for selection of lathe. *Journal of Applied Engineering Science*, 20(4), 2022: 1355-1365. https://doi.org/10.5937/jaes0-39335

[18] S. Chakraborty, A.K. Saha, Selection of forklift unit for transport handling using integrated MCDM under neutrosophic environment. *Facta Universitatis - Series: Mechanical Engineering*, 22(2), 2024: 235-256. https://doi.org/10.22190/FUME220620039C

[19] J. Qi, Machine Tool Selection Model Based on Fuzzy MCDM Approach. *2010 International Conference on Intelligent Control and Information Processing*, Dalian, China, 2010, 282-285. https://doi.org/10.1109/ICICIP.2010.5564228

[20] Y. Ersoy, Green Machine Selection in a Manufacturing Company Using TOPSIS Method. *Academia Journal of Nature and Human Sciences*, 6(1), 2020: 45-56.

[21] N.Yalci, N. Uncu, Applying EDAS as an applicable MCDM method for industrial robot selection. *Sigma Journal of Engineering and Natural Sciences*, 37(3), 2019: 779-796.

[22] S. Alpay, M. Iphar, Equipment selection based on two different fuzzy multi criteria decision making methods: Fuzzy TOPSIS and fuzzy VIKOR. *De Gruyter*, 10(1), 2018: 661-677. https://doi.org/10.1515/geo-2018-0053

[23] L.A.P. Dominguez, E.Z. Borroel, O.E.I. Quezada, D. Ortiz-Munoz, A. Najera-Acosta, CODAS, TOPSIS, and AHP Methods Application for Machine Selection. *Journal of Computational and Cognitive Engineering*, 2(4), 2023: 322–330. https://doi.org/10.47852/bonviewJCCE3202428

[24] Z. Štirbanović, D. Stanujkić, I. Miljanović, D. Milanović, Application of MCDM methods for flotation machine selection. *Minerals Engineering*, 137, 2019: 140-146. https://doi.org/10.1016/j.mineng.2019.04.014

[25] A.T. Nguyen, H.A. Bui, A novel multi-criteria decision making procedure for saw machine selection in the mechanical machining. *Hanoi University of Industry Journal of Science and Technology*, 59(3), 2023: 100-107. https://doi.org/10.57001/huih5804.2023.117

[26] A. Ozgen, G. Tuzkaya, U.R. Tuzkaya, D. Ozgen, A Multi-Criteria Decision Making Approach for Machine Tool Selection Problem in a Fuzzy Environment. *International Journal of Computational Intelligence Systems*, 4(4), 2021: 431-445. https://doi.org/10.1080/18756891.2011.9727802

[27] A. Sarkar, S.C. Panja, D. Das, B. Sarkar, Developing an efficient decision support system for non-traditional machine selection: an application of MOORA and MOOSRA. *Production & Manufacturing Research*, 3(1), 2015: 324-342. https://doi.org/10.1080/21693277.2014.895688

[28] S.S. Goswami, D.K. Behera, Developing Fuzzy-AHP-Integrated Hybrid MCDM System of COPRAS-ARAS for Solving an Industrial Robot Selection Problem. *International Journal of Decision Support System Technology*, 15(1), 2023: 1-38. https://doi.org/10.4018/IJDSST.324599

[29] P. Venkateswarlu, B.D. Sarma, Selection of Equipment by Using Saw and Vikor Methods. *International Journal of Engineering Research and Application,* 6(11), 2016: 61-68.

[30] S.S. Goswami, D.K. Behera, Solving Material Handling Equipment Selection Problems in an Industry with the Help of Entropy Integrated COPRAS and ARAS MCDM techniques. *Process Integration and Optimization for Sustainability,* 5, 2021: 947–973. https://doi.org/10.1007/s41660-021-00192-5

[31] M. Ozcalici, Integrating queue theory and multi-criteria decision-making tools for selecting roll-over car washing machine. *Operations Research and Decisions*, 23(2), 2023: 99-119. https://doi.org/10.37190/ord230206

[32] D. D. Trung, B. Dudić, D. V. Duc, N. H.Son, A. Ašonja, Comparison of MCDM methods effectiveness in the selection of plastic injection molding machines. *Teknomekanik*, 7(1), 2024: 1-19. https://doi.org/10.24036/teknomekanik.v7i1.29272

[33] S.S. Goswami, D.K. Behera, A. Afzal, A. Razak Kaladgi, S. A. Khan, P. Rajendran, R. Subbiah, M. Asif, Analysis of a Robot Selection Problem Using Two Newly Developed Hybrid MCDM Models of TOPSIS-ARAS and COPRAS-ARAS. *Symmetry*, 13(8), 2021: 1331. https://doi.org/10.3390/sym13081331

[34] P. Aazagreyir, P. Appiahene, O. Appiah, S. Boateng, Comparative analysis of fuzzy multi-criteria decision-making methods for quality of service-based web service selection. *IAES International Journal of Artificial Intelligence*, 13(2), 2024: 1408-1419. https://doi.org/10.11591/ijai.v13.i2.pp1408-1419

[35] T.V. Dua, Combination of symmetry point of criterion, compromise ranking of alternatives from distance to ideal solution and collaborative unbiased rank list integration methods for woodworking machinery selection for small business in Vietnam. *EUREKA: Physics and Engineering*, 2023(2), 2023: 83-96. https://doi.org/10.21303/2461-4262.2023.002763

[36] T.V. Dua, Forklift selection by multi-criteria decision-making methods. *Eastern-European Journal of Enterprise Technologies*, 5(3), 2023: 95–101. https://doi.org/10.15587/1729-4061.2023.285791

[37] H.X. Thinh, T.V. Dua, Enhancing Understanding of Changes in Solution Rankings with Variations in User Coefficients in AROMAN Method. *International Journal of Mechanical Engineering and Robotics Research*, 13(3), 2024: 354-361. https://doi.org/10.18178/ijmerr.13.3.354-361

[38] N.H. Son, T.T. Hieu, N.M. Thang, H.N. Tan, N. T. Can, P.T. Thao, N.C. Bao, Choosing the best machine tool in mechanical manufacturing. *EUREKA: Physics and Engineering*, 2023(2), 2023: 97-109. https://doi.org/10.21303/2461-4262.2023.002771

[39] A. Puška, Ž. Stevic, D. Pamuča, Evaluation and selection of healthcare waste incinerators using extended sustainability criteria and multi-criteria analysis methods. *Environment, Development and Sustainability*, 24, 2022: 11195–11225. https://doi.org/10.1007/s10668-021-01902-2

[40] M. Yazdani, P. Zaraté, E.K. Zavadskas, Z. Turskis, A Combined Compromise Solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision, Emerald*, 57(9), 2019: 2501-2519. https://doi.org/10.1108/MD-05-2017-0458

[41] E. Haktanır, C. Kahraman, Integrated AHP & TOPSIS methodology using intuitionistic Z-numbers: An application on hydrogen storage technology selection. *Expert Systems with Applications*, 239, 2024: 122382. https://doi.org/10.1016/j.eswa.2023.122382

[42] A. Sotoudeh-Anvari, Root Assessment Method (RAM): A novel multi-criteria decision making method and its applications in sustainability challenges. *Journal of Cleaner Production*, 423, 2023: 138695. https://doi.org/10.1016/j.jclepro.2023.138695

[43] S. Bošković, L. Švadlenka, S. Jovčić, M. Dobrodolac, V. Simić, N. Bačanin, An Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN) – A Case Study of the Electric Vehicle Selection Problem. *IEEE Access*, 11, 2023: 39496-39507. https://doi.org/10.1109/ACCESS.2023.3265818

[44] A.-T. Nguyen, Combining FUCA, CURLI, and Weighting Methods in the Decision-Making of Selecting Technical Products. *Engineering, Technology & Applied Science Research*, 13(4), 2023: 11222-11229. https://doi.org/10.48084/etasr.6015