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CONVERGENCE STRATEGIES FOR OPTIMIZING ANTENNA SELECTION IN A COMMUNICATION SYSTEM: A COMPLEX LINEAR DIOPHANTINE FUZZY SOFT SET APPROACH

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Abstract:
The need to grow in a secure and tranquil environment demands the efforts of an armed force, and only with a strong-armed force can a country ensure its national security. In military activities, communication devices are widely used to confuse enemies' radars or communications to abandon their strategies and execute planned actions. The range of communication devices depends mainly on the antennas used. Army sustainability goals are to upgrade the effectiveness of the mission, reduce army environmental impact, build green sustainable structures, and attain the energy level independence that improves the continuity of operations which are indispensable to the mission. The primary goal of this paper is to present an innovative mathematical model for selecting pertinent antennae in communication devices using an innovative idea called a Complex Linear Diophantine Fuzzy Soft set based on the various attributes by incorporating decision-making techniques. Also, some of its beneficial operations such as Complement, AND, OR, Extended Union, and Extended Intersection, are presented in concert with the properties and theorems to apprise the viability of the proposed paper. This concept is more applicable and necessary to assess real-life situations using mathematical modeling.

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1. INTRODUCTION

A military, also known collectively as armed forces, is a meticulously organized entity designed mainly for warfare. It is authorized and administrated by a sovereign state, with the members distinguished by their distinctive military uniform. The branches of a military force include the Air Force, Space Force, army, navy, marines, and Coast Guard. The indispensable role of the armed forces is to maintain peace and enhance the overall welfare of the country. The achievement of all entities depends on effective communication, and hence its significance is indicated by the effective functioning of everyday operations. As a result, members of the armed forces constantly encourage the process of obtaining the required clarification and the entire process between subordinates and leaders in resolving issues through efficient communication because a lack of solid interaction leads to an unsuccessful mission.

A clear and concise exchange of information for accomplishing the mission includes eight essential elements such as source, message, channel, receiver, feedback, environment, context, and interference. A signal that is responsible for the army's entire system of communication dwells in the extremities of the antenna. To execute an entire operation according to the specifications of the armed forces, antennas are extremely important [1-3]. The primary goal of this paper is to present an innovative mathematical

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model for selecting pertinent antennae in communication devices.

1.1 Relevant Literature

A majority of real-world problems in medicine, forecasting, engineering, agriculture, etc., encompass imprecise data, and addressing these problems demands the use of some computational techniques based on ambiguity and uncertainty. Many of the issues are humanistic and, therefore, subjective. In the last few decades, an abundance of theories to assist these kinds of systems has been addressed by many researchers. Some of the theories encompass the following:

1. Fuzzy Set ($\mathbb{FS}$) initiated by Zadeh [4] is a valuable tool for describing circumstances involving imprecise data.
2. Intuitionistic Fuzzy Set ($\mathbb{IFS}$) established by Attanasov [5] as a protraction of $\mathbb{FS}$, delineated by Membership Score ($\mathbb{MS}$) and Non-Membership Score ($\mathbb{NMS}$) alongside the limitation that $0 \leq \mathbb{MS} + \mathbb{NMS} \leq 1$.
3. Yager [6] instigated the conceptualization of the Pythagorean Fuzzy Set ($\mathbb{PFS}$) as a prolongation of $\mathbb{IFS}$, alongside the limitation that $0 \leq (\mathbb{MS})^2 + (\mathbb{NMS})^2 \leq 1$.
4. To deal with the real-world problems that $\mathbb{PFS}$ cannot address is handled by a perception of q-Rung Orthopair Fuzzy Set (q- $\mathbb{ROFS}$) which was introduced by Yager [7] alongside the restriction that $0 \leq (\mathbb{MS})^q + (\mathbb{NMS})^q \leq 1$.

Riaz and Hashmi [8] suggested the thought of a Linear Diophantine Fuzzy Set ($\mathbb{LDFS}$) by merely including the reference parameter. Many researchers have worked on the development of $\mathbb{FS}$ theory with applications in various fields [9-13].

However, the aforementioned theories, have an inherent restriction that is insufficient for handling problems involving parametrization tools.

Molodtsov [14] pioneered an abstraction of Soft Set ($\mathbb{SS}$) theory as mathematical equipment that is free of the above-mentioned restriction. In many of the circumstances, $\mathbb{MS}$, $\mathbb{NMS}$ and reference parameters could be perceived as Complex Valued ($\mathbb{CV}$) to address this research gap, Ramot at al. [15] offered an intellection of the Complex Fuzzy set ($\mathbb{CFS}$) signalized by Complex-Valued Membership Score ($\mathbb{CVMS}$) $\Phi(\mathrm{x})e^{i2\pi w_{\Phi}(x)}$ where, $\Phi(\mathrm{x})$ indicate Amplitude Term ($\mathbb{AT}$) and $w_{\Phi}(\mathrm{x})$ indicate phase term ($\mathbb{PT}$). To enlarge the space, Kamaci [16] addressed the impression of a Complex Linear Diophantine Fuzzy set ($\mathbb{CLDFS}$) that is specialized by $\mathbb{CVMS}$, Complex-Valued Non-Membership Score ($\mathbb{CVNMS}$) and Complex Valued Reference Parameters. Retaining this advantage, the conceptualization of Complex Linear Diophantine Fuzzy Soft Set ($\mathbb{CLDFSS}$) is instigated with the advantage of confronting real-world problems that comprise the parametrization tool. The development of $\mathbb{FS}$ theory is discussed in [17-21].

Several important studies have made a substantial contribution to the understanding and ability to optimize communication protocols by utilizing fuzzy logic and decision-making frameworks in the investigation of secure military communication. Tyagi and Singal [22] demonstrated that fuzzy logic control systems could improve operational effectiveness and decision-making precision by focusing on their application to military equipment. To optimize communication channels in difficult situations, Qiu et al. [23] created an algorithm for military communication mode optimization based on fuzzy multi-objective decision-making. To overcome the inherent uncertainties and ambiguities in complex military environments, Hanratty et al. [24] presented a fuzzy-based strategy to help decision-making. Dockery [25] examined the fuzzy design of information systems used in the military, emphasizing advancements in the resilience and flexibility of information management architectures. Deveci et al. [26] emphasized the theory's adaptability and use in a range of defense contexts by thoroughly examining the fuzzy set theory's military applications. Wang et al. [27] applied probability hesitant intuitionistic fuzzy approaches to assess the efficacy of constellation satellite communication systems, highlighting the significance of consensus and consistency in communication protocols in the face of uncertainty.

1.1.1 Advantages and Limitations of the Existing Methodologies

The possible range of the already existing $\mathbb{FS}$ theory is broadened to handle real-world problems involving parametrization tools in the complex plane. To concurrently capture the ambiguity and periodicity of the problems in a complex plane, a conceptualization of $\mathbb{CLDFSS}$ can be implemented. The polar formulation is employed to demonstrate the beneficial effects of the periodic traits and the parametrization tool inherent in $\mathbb{CV}$ functions. The Advantages and Limitations of the existing methodology are inferred in Table 1.

Table 1. The Advantages and Limitations of the existing methodology

Mathematical tool	Advantages	Limitations
$\mathbb{FS}$ [4]	Handle problems with imprecision and ambiguity in the information set with the help of Membership Function ($\mathbb{MF}$) by providing $\mathbb{MS}$ between truth and falseness	There is no Non-membership Function ($\mathbb{NMF}$) which is crucial for elements of an information set
$\mathbb{IFS}$ [5]	Both $\mathbb{MF}$ and $\mathbb{NMF}$ exist	The sum of $\mathbb{MS}$ and $\mathbb{NMS}$ is constrained to 1
$\mathbb{PFS}$ [6]	The assessment space for membership and non-membership is expanded in comparison to $\mathbb{IFS}$	The sum of the square of $\mathbb{MS}$ and $\mathbb{NMS}$ is constrained to 1
$\mathbb{LDFS}$ [8]	The assessment space for membership and non-membership is expanded with the addition of reference parameter comparison to $\mathbb{PFS}$	Inadequate in handling problems with the parametrization tool
$\mathbb{SS}$ [14]	The parametrization tool is accessible	It is incapable of representing the fuzziness of the parameters in a 2D frame
$\mathbb{CFS}$ [15]	Address fuzzy information in a 2D frame of reference	Inadequacy of processing $\mathbb{LDF}$ information
$\mathbb{CLDFS}$ [16]	Address $\mathbb{LDF}$ v information in a 2D frame of reference	Inadequate in handling problems involving $\mathbb{LDF}$ information in 2D frame of reference with parametrization tool

Numerous studies have highlighted the importance of $\mathbb{FS}$ theories in improving the security, effectiveness, and dependability of military communication systems [28-30].

1.2 Research Problem

Since the previously mentioned theories are effectively applied in diverse settings, they show certain inherent difficulties when addressing practical issues about parametrization tools within a two-dimensional frame of reference. To address real-world issues concerning the parametrization tool in a 2D frame of reference, Jayakumar et al. [31] developed the Complex Linear Diophantine Fuzzy Soft set ($\mathbb{CLDFSS}$) framework.

Furthermore, identifying antennas that can operate efficiently under a variety of challenging conditions is a major problem in military communications. The choice of suitable antennas, which is determined by several variables such as mission requirements and ambient circumstances, is critical to the performance and dependability of communication systems.

1.3 Objectives and Layout of the Manuscript

The Identified problem can be considered a multi-observer decision-making problem, in which the result of possession depends on the collection of inputs from various observers. Therefore, in the context of $\mathbb{CLDFSS}$ environment, the intended objective of this paper has been split into three parts:

1. A perception of $\mathbb{CLDFSS}$ is put in place as an amalgamation of $\mathbb{CLDFS}$ and $\mathbb{SS}$.
2. Beneficial operational laws of $\mathbb{CLDFSS}$ is investigated alongside some examples and theorems.
3. A mathematical model is presented for selecting pertinent antennae in communication devices using an innovative idea called $\mathbb{CLDFSS}$ based on the various attributes by incorporating Multi Criteria Decision Making ($\mathbb{MCDM}$) technique.

The manuscript will be laid out in the following fashion. "Section 2" contains specific definitions of existing paradigms. In "Section 3", the beneficial operational laws of $\mathbb{CLDFSS}$ are instigated and illustrated through examples and theorems. "Section 4" encompasses a mathematical simulation of $\mathbb{CLDFSS}$ for a pertinent antennae selection in a communication device to attain army sustainability goals. In "Section 5", a comparative assessment of the proposed mathematical modeling with the existing theories is described.

2. THEORETICAL FOUNDATIONS

This section emphasizes the rationale of $\mathbb{FS}$, $\mathbb{SS}$, $\mathbb{IFS}$, $\mathbb{IFSS}$, $\mathbb{CLDFS}$, $\mathbb{CLDFSS}$ and $\mathbb{ACR}$.

Throughout this article, $\mathbb{K}$ signifies the universal set ($\mathbb{US}$) and $\mathbb{C}$ signifies the Attribute Set ($\mathbb{AS}$).

Definition 2.1. [4]: The Fuzzy set interpreted on $\mathbb{K}$ by:

$$\mathrm{D} = \{\langle k_m, \Phi_D(k_m)\rangle : k_m \in \mathbb{K}\},$$

where $\Phi_D : \mathbb{K} \to [0,1]$ and it is known as $\mathbb{MS}$ of $k_m \in \mathbb{K}$ to the set D.

Definition 2.2. [14]: A pair $(\mathrm{X}, \mathbb{C})$ is articulated to be a Soft set over $\mathbb{K}$, where X is a mapping specified as:

$$\mathrm{X} : \mathbb{C} \to P(\mathbb{K}),$$

where $P(\mathbb{K})$ is the power set of $\mathbb{K}$.

Definition 2.3. [5]: The Intuitionistic Fuzzy set is an object that has the following structure:

$$\mathrm{Q} = \{(k_m, \langle \Phi_Q(k_m), \Psi_Q(k_m)\rangle) : k_m \in \mathbb{K}\},$$

where $\Phi_Q(k_m), \Psi_Q(k_m) \in [0,1]$ and it is known as $\mathbb{MS}$ and $\mathbb{NMS}$ respectively to the set Q with regard to the specification that $0 \le \Phi_Q(k_m) + \Psi_Q(k_m) \le 1$.

Definition 2.4. [32]: A pair $(\mathrm{G}, \mathbb{C})$ is articulated to be an Intuitionistic Fuzzy Soft set over $\mathbb{K}$, where G is a mapping interpreted as:

$$\mathrm{G} : \mathbb{C} \to \mathbb{IFS}(\mathbb{K}),$$

for $\mathrm{c} \in \mathbb{C}$ such that $\mathrm{G(c)} = \phi$ if $\mathrm{c} \notin \mathbb{C}$ and $\mathbb{IFS}(\mathbb{K})$ be the collection of all Intuitionistic Fuzzy Subsets of $\mathbb{K}$.

Definition 2.5. [16]: The Complex linear diophantine Fuzzy set is an object that has the following structure [10]:

$$\mathrm{J} = \{(k_m, \langle \Phi_J(k_m) e^{i2\pi\left(w_{\Phi_J}(k_m)\right)}, \Psi_J(k_m) e^{i2\pi\left(w_{\Psi_J}(k_m)\right)}\rangle, \langle \zeta_J^m e^{i2\pi\left(w_{\zeta_J^m}\right)}, \eta_J^m e^{i2\pi\left(w_{\eta_J^m}\right)}\rangle) : k_m \in \mathbb{K}\},$$

where: $\Phi_J(k_m) e^{i2\pi\left(w_{\Phi_J}(k_m)\right)}$, $\Psi_J(k_m) e^{i2\pi\left(w_{\Psi_J}(k_m)\right)}$, $\zeta_J^m e^{i2\pi\left(w_{\zeta_J^m}\right)}$ and $\eta_J^m e^{i2\pi\left(w_{\eta_J^m}\right)}$ signifies the class of $\mathbb{CVMS}$, $\mathbb{CVNMS}$, complex-valued reference parameter correlate with the $\mathbb{MS}(\mathbb{RPMS})$ and complex-valued reference parameter correlate with the $\mathbb{NMS}(\mathbb{RPNMS})$ respectively of $k_m \in \mathbb{K}$ to the set J with regard to the specification that $0 \le \zeta_J^m + \eta_J^m \le 1$, $0 \le \zeta_J^m \Phi_J(k_m) + \eta_J^m \Psi_J(k_m) \le 1$, $0 \le w_{\zeta_J^m} + w_{\eta_J^m} \le 1$, $0 \le w_{\zeta_J^m} w_{\Phi_J}(k_m) + w_{\eta_J^m} w_{\Psi_J}(k_m) \le 1$.

Definition 2.6. [31]: A pair $(\mathbb{W}, \mathbb{C})$ is articulated to be a Complex Linear Diophantine Fuzzy Soft set over $\mathbb{K}$, where $\mathbb{W}$ is a mapping interpreted as

$$\mathbb{W} : \mathbb{C} \to \mathbb{CLDFSU}(\mathbb{K}),$$

for $\mathrm{c} \in \mathbb{C}, \mathbb{W}(\mathrm{c}) \in \mathbb{CLDFSU}(\mathbb{K})$ and it can be expressed as:

$$\mathbb{W}(c) = \{(k_m, \langle \Phi_{\mathbb{W}(c)}(k_m) e^{i2\pi\left(w_{\Phi_{\mathbb{W}(c)}}(k_m)\right)}, \Psi_{\mathbb{W}(c)}(k_m) e^{i2\pi\left(w_{\Psi_{\mathbb{W}(c)}}(k_m)\right)}\rangle, \langle \zeta_{\mathbb{W}(c)}^m e^{i2\pi\left(w_{\zeta_{\mathbb{W}(c)}^m}\right)}, \eta_{\mathbb{W}(c)}^m e^{i2\pi\left(w_{\eta_{\mathbb{W}(c)}^m}\right)}\rangle) : k_m \in \mathbb{K}\},$$

such that $\mathbb{W}(c) = \phi$ if $\mathrm{c} \notin \mathbb{C}$ and $\mathbb{CLDFSU}(\mathbb{K})$ be the collection of all Complex Linear Diophantine Fuzzy Subsets of $\mathbb{K}$. It can also be expressed as:

$$\mathbb{W}(\mathrm{c}) = \{(k_m, \langle(\Phi_{\mathbb{W}(c)}(k_m), w_{\Phi_{\mathbb{W}(c)}}(k_m)), (\Psi_{\mathbb{W}(c)}(k_m), w_{\Psi_{\mathbb{W}(c)}}(k_m)\rangle \langle(\zeta_{\mathbb{W}(c)}^m, w_{\zeta_{\mathbb{W}(c)}^m}), (\eta_{\mathbb{W}(c)}^m, w_{\eta_{\mathbb{W}(c)}^m})\rangle) : k_m \in \mathbb{K}\}.$$

Definition 2.7. [31]: Let $\mathrm{U} = (\langle(\Phi_U, w_{\Phi_U}), (\Psi_U, w_{\Psi_U})\rangle, \langle(\zeta_U, w_{\zeta_U}), (\eta_U, w_{\eta_U})\rangle)$ be a Complex Linear Diophantine Fuzzy Soft Number($\mathbb{CLDFSN}$). Then:

$$\mathbb{ACR}(\mathrm{U}) = \frac{1}{4}\left[\frac{(\Phi_U + \Psi_U)}{2} + \frac{(w_{\Phi_U} + w_{\Psi_U})}{2} + (\zeta_U + \eta_U) + (w_{\zeta_U} + w_{\eta_U})\right], \tag{1}$$

where $\mathbb{ACR}(\mathrm{U}) \in [0,1]$ and it is particularized as the Accuracy function of U.

3. FUNDAMENTAL OPERATIONS OF COMPLEX LINEAR DIOPHANTINE FUZZY SOFT SET

Definition 3.1. Considering the Complex Linear Diophantine Fuzzy Soft set $(\mathbb{W}, \mathbb{C})$, the complement of $(\mathbb{W}, \mathbb{C})$ is designated as $(\mathbb{W}, \mathbb{C})^c = (\mathbb{W}^c)$, where

$\mathbb{W}^c: \mathbb{C} \to \mathbb{CLDFSU}(\mathbb{K})$ signifies the mapping particularized by:

$$\mathbb{W}(c) = \{(k_m, \langle \Psi_{\mathbb{W}(c)}(k_m) e^{i2\pi\left(w_{\Psi_{\mathbb{W}(c)}}(k_m)\right)}, \Phi_{\mathbb{W}(c)}(k_m) e^{i2\pi\left(w_{\Phi_{\mathbb{W}(c)}}(k_m)\right)} \rangle \langle \eta^m_{\mathbb{W}(c)} e^{i2\pi\left(w_{\eta^m_{\mathbb{W}(c)}}\right)}, \zeta^m_{\mathbb{W}(c)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{W}(c)}}\right)} \rangle): k_m \in \mathbb{K}\}.$$

Definition 3.2. Let $(\mathbb{X}, \mathbb{D})$ and $(\mathbb{V}, \mathbb{E})$ be two $\mathbb{CLDFSS}$, where $\mathbb{D}$ and $\mathbb{E}$ are subsets of $\mathbb{C}$. Then $(\mathbb{X}, \mathbb{D})$ AND $(V, \mathbb{E})$ is specified as $(\mathbb{X}, \mathbb{D}) \wedge (\mathbb{V}, \mathbb{E}) = (\mathbb{P}, (\mathbb{D} \times \mathbb{E}))$ (i.e.), $(\mathbb{P}, (d, e)) = (\mathbb{X}, \mathbb{D}) \cap (\mathbb{V}, \mathbb{E}), \forall (d, e) \in \mathbb{D} \times \mathbb{E}$

$$\Rightarrow \mathbb{P}(d, e) = \{(k_m, \langle \Phi_{\mathbb{P}(d,e)}(k_m) e^{i2\pi\left(w_{\Phi_{\mathbb{P}(d,e)}}(k_m)\right)}, \Psi_{\mathbb{P}(d,e)}(k_m) e^{i2\pi\left(w_{\Psi_{\mathbb{P}(d,e)}}(k_m)\right)} \rangle, \langle \zeta^m_{\mathbb{P}(d,e)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{P}}(d,e)}\right)}, \eta^m_{\mathbb{P}(d,e)} e^{i2\pi\left(w_{\eta^m_{\mathbb{P}}(d,e)}\right)} \rangle): k_m \in \mathbb{K}\}, \quad (2)$$

where:

$$\left\{ \begin{array}{c} \Phi_{\mathbb{P}(d,e)}(k_m) = \min\{\Phi_{\mathbb{X}(d)}(k_m), \Phi_{\mathbb{V}(e)}(k_m)\} \\ w_{\Phi_{\mathbb{P}(d,e)}}(k_m) = \min\left\{w_{\Phi_{\mathbb{X}(d)}}(k_m), w_{\Phi_{\mathbb{V}(e)}}(k_m)\right\} \\ \Psi_{\mathbb{P}(d,e)}(k_m) = \max\{\Psi_{\mathbb{X}(d)}(k_m), \Psi_{\mathbb{V}(e)}(k_m)\} \\ w_{\Psi_{\mathbb{P}(d,e)}}(k_m) = \max\left\{w_{\Psi_{\mathbb{X}(d)}}(k_m), w_{\Psi_{\mathbb{V}(e)}}(k_m)\right\} \\ \zeta^m_{\mathbb{P}(d,e)} = \min\{\zeta^m_{\mathbb{X}(d)}, \zeta^m_{\mathbb{V}(e)}\} \\ w_{\zeta^m_{\mathbb{P}}(d,e)} = \min\left\{w_{\zeta^m_{\mathbb{X}(d)}}, w_{\zeta^m_{\mathbb{V}(e)}}\right\} \\ \eta^m_{\mathbb{P}(d,e)} = \max\{\eta^m_{\mathbb{X}(d)}, \eta^m_{\mathbb{V}(e)}\} \\ w_{\eta^m_{\mathbb{P}(d,e)}} = \max\left\{w_{\eta^m_{\mathbb{X}(d)}}, w_{\eta^m_{\mathbb{V}(e)}}\right\} \end{array} \right\}_{\{m=1,2,\ldots,s\}}.$$

Example 3.3. Consider two $\mathbb{CLDFSS}(\mathbb{X}, \mathbb{D})$ and $(\mathbb{V}, \mathbb{E})$ in a tabular representation as specified in Tables 2 and 3. Then $(\mathbb{X}, \mathbb{D}) \wedge (\mathbb{V}, \mathbb{E})$ is given in Table 4.

Table 2. $\mathbb{CLDFSS}(\mathbb{X}, \mathbb{D})$

$(\mathbb{X}, \mathbb{D})$	d_1	d_2
k_1	⟨(0.6,0.7), (0.3,0.2)⟩, ⟨(0.8,0.7), (0.1,0.3)⟩	⟨(0.5,0.6), (0.4,0.4)⟩, ⟨(0.7,0.8), (0.2,0.2)⟩
k_2	⟨(0.7,0.7), (0.2,0.3)⟩, ⟨(0.7,0.8), (0.3,0.2)⟩	⟨(0.9,0.8), (0.1,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩
k_3	⟨(0.5,0.7), (0.3,0.2)⟩, ⟨(0.6,0.7), (0.4,0.3)⟩	⟨(0.6,0.5), (0.3,0.4)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩

Table 3. $\mathbb{CLDFSS}(\mathbb{V}, \mathbb{E})$

$(\mathbb{V}, \mathbb{E})$	e_1	e_2
k_1	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩	⟨(0.6,0.7), (0.3,0.3)⟩, ⟨(0.5,0.7), (0.4,0.3)⟩
k_2	⟨(0.7,0.6), (0.2,0.2)⟩, ⟨(0.6,0.6), (0.3,0.4)⟩	⟨(0.5,0.7), (0.3,0.2)⟩, ⟨(0.8,0.7), (0.2,0.3)⟩
k_3	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.7,0.6), (0.3,0.3)⟩	⟨(0.6,0.5), (0.3,0.2)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩

Table 4. $(\mathbb{X}, \mathbb{D}) \wedge (\mathbb{V}, \mathbb{E})$

$(\mathbb{X}, \mathbb{D}) \wedge (\mathbb{V}, \mathbb{E})$	(d_1, e_1)	(d_1, e_2)	(d_2, e_1)	(d_2, e_2)
k_1	⟨(0.6,0.7), (0.3,0.2)⟩, ⟨(0.8,0.7), (0.1,0.3)⟩	⟨(0.6,0.7), (0.3,0.3)⟩, ⟨(0.5,0.7), (0.4,0.3)⟩	⟨(0.5,0.6), (0.4,0.4)⟩, ⟨(0.7,0.8), (0.2,0.2)⟩	⟨(0.5,0.6), (0.4,0.4)⟩, ⟨(0.5,0.7), (0.4,0.3)⟩
k_2	⟨(0.7,0.6), (0.2,0.3)⟩, ⟨(0.6,0.6), (0.3,0.4)⟩	⟨(0.5,0.7), (0.3,0.3)⟩, ⟨(0.7,0.7), (0.3,0.3)⟩	⟨(0.7,0.6), (0.2,0.2)⟩, ⟨(0.6,0.6), (0.3,0.4)⟩	⟨(0.5,0.7), (0.3,0.2)⟩, ⟨(0.8,0.7), (0.2,0.3)⟩
k_3	⟨(0.5,0.7), (0.3,0.2)⟩, ⟨(0.6,0.6), (0.4,0.3)⟩	⟨(0.5,0.5), (0.3,0.2)⟩, ⟨(0.6,0.7), (0.4,0.3)⟩	⟨(0.6,0.5), (0.3,0.4)⟩, ⟨(0.7,0.6), (0.3,0.3)⟩	⟨(0.6,0.5), (0.3,0.4)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩

Definition 3.4. Let $(\mathbb{X}, \mathbb{D})$ and $(\mathbb{V}, \mathbb{E})$ be two $\mathbb{CLDFSS}$, where $\mathbb{D}$ and $\mathbb{E}$ are subsets of $\mathbb{C}$. Then $(\mathbb{X}, \mathbb{D}) OR (\mathbb{V}, \mathbb{E})$ is specified as $(\mathbb{X}, \mathbb{D}) \vee (\mathbb{V}, \mathbb{E}) = (\mathbb{R}, (\mathbb{D} \times \mathbb{E}))$.
(i.e), $(\mathbb{R}, (d, e)) = (\mathbb{X}, \mathbb{D}) \cup (\mathbb{V}, \mathbb{E}), \forall (d, e) \in \mathbb{D} \times \mathbb{E}$

$\Rightarrow \mathbb{R}(\boldsymbol{d},\boldsymbol{e}) = \{(\boldsymbol{k_m},$

$\langle \boldsymbol{\Phi}_{\mathbb{R}(d,e)}(\boldsymbol{k_m})\boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Phi_{\mathbb{R}(d,e)}}(\boldsymbol{k_m})\right)}, \boldsymbol{\Psi}_{\mathbb{R}(d,e)}(\boldsymbol{k_m})\boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Psi_{\mathbb{R}(d,e)}}(\boldsymbol{k_m})\right)}\rangle,$

$\langle \boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{R}(d,e)}\boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\zeta^m_{\mathbb{R}(d,e)}}\right)}, \boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{R}(d,e)}\boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\eta^m_{\mathbb{R}(d,e)}}\right)}\rangle): \boldsymbol{k_m} \in \mathbb{K}\},$ (3)

where:

$$\left\{\begin{array}{c} \Phi_{\mathbb{R}(d,e)}e(k_m) = \max\{\Phi_{\mathbb{X}(d)}(k_m), \Phi_{\mathbb{V}(e)}(k_m)\} \\ w_{\Phi_{\mathbb{R}(d,e)}}(k_m) = \max\left\{w_{\Phi_{\mathbb{X}(d)}}(k_m), w_{\Phi_{\mathbb{V}(e)}}(k_m)\right\} \\ \Psi_{\mathbb{R}(d,e)}(k_m) = \min\{\Psi_{\mathbb{X}(d)}(k_m), \Psi_{\mathbb{V}(e)}(k_m)\} \\ w_{\Psi_{\mathbb{R}(d,e)}}(k_m) = \min\left\{w_{\Psi_{\mathbb{X}(d)}}(k_m), w_{\Psi_{\mathbb{V}(e)}}(k_m)\right\} \\ \zeta^m_{\mathbb{R}(d,e)} = \max\{\zeta_{\mathbb{X}(d)}, \zeta^m_{\mathbb{V}(e)}\} \\ w_{\zeta^m_{\mathbb{R}}(d,e)} = \max\left\{w_{\zeta^m_{\mathbb{X}(d)}}, w_{\zeta^m_{\mathbb{V}(e)}}\right\} \\ \eta^m_{\mathbb{R}(d,e)} = \min\{\eta^m_{\mathbb{X}(d)}, \eta^m_{\mathbb{V}(e)}\} \\ w_{\eta^m_{\mathbb{R}(d,e)}} = \min\left\{w_{\eta^m_{\mathbb{X}(d)}}, w_{\eta^m_{\mathbb{V}(e)}}\right\} \end{array}\right\}_{\{m=1,2,\dots,s\}}.$$

Example 3.5. Consider two $\mathbb{CLDFSS}(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ as given in Example 1. Then $(\mathbb{X},\mathbb{D}) \vee (\mathbb{V},\mathbb{E})$ is given in Table 5.

Table 5. $(\mathbb{X},\mathbb{D}) \vee (\mathbb{V},\mathbb{E})$

$(\mathbb{X},\mathbb{D}) \vee (\mathbb{V},\mathbb{E})$	(d_1,e_1)	(d_1,e_2)	(d_2,e_1)	(d_2,e_2)
k_1	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩	⟨(0.6,0.7), (0.3,0.2)⟩, ⟨(0.8,0.7), (0.1,0.3)⟩	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩	⟨(0.6,0.7), (0.3,0.3)⟩, ⟨(0.7,0.8), (0.2,0.2)⟩
k_2	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.7,0.8), (0.3,0.2)⟩	⟨(0.7,0.7), (0.2,0.2)⟩ ⟨(0.8,0.8), (0.2,0.2)⟩	⟨(0.9,0.8), (0.1,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩	⟨(0.9,0.8), (0.1,0.2)⟩, ⟨(0.8,0.8), (0.1,0.2)⟩
k_3	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.7,0.7), (0.3,0.3)⟩	⟨(0.6,0.7), (0.3,0.2)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩	⟨(0.7,0.7), (0.2,0.2)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩	⟨(0.6,0.5), (0.3,0.2)⟩, ⟨(0.8,0.9), (0.1,0.1)⟩

Theorem 3.6. Consider two $\mathbb{CLDFSS}(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$. Then

1. $((\mathbb{X},\mathbb{D}) \wedge (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \vee (\mathbb{V},\mathbb{E})^c$
2. $((\mathbb{X},\mathbb{D}) \vee (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \wedge (\mathbb{V},\mathbb{E})^c$.

Proof. Let us suppose that $(\mathbb{X},\mathbb{D}) \wedge (\mathbb{V},\mathbb{E}) = (\mathbb{P},\mathbb{D}\times\mathbb{E})$, where $(\mathbb{P},(d,e)) = (\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E})$.

(i.e.) , $\mathbb{P}(d,e) = \{(k_m,$

$$\left\langle\begin{array}{l} \min\left\{\Phi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}(d)}}(k_m)\right)}, \Phi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}(e)}}(k_m)\right)}\right\} \\ \max\left\{\Psi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}(d)}}(k_m)\right)}, \Psi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}(e)}}(k_m)\right)}\right\} \end{array}\right\rangle$$

$$\left\langle\begin{array}{l} \min\left\{\zeta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}(d)}}\right)}, \zeta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}(e)}}\right)}\right\} \\ \max\left\{\eta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\eta^m_{\mathbb{X}(d)}}\right)}, \eta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\eta^m_{\mathbb{V}(e)}}\right)}\right\} \end{array}\right\rangle): k_m \in \mathbb{K}\}, \quad (4)$$

$\forall(\mathrm{d},\mathrm{e}) \in (\mathbb{D}\times\mathbb{E})$. Now, $((\mathbb{X},\mathbb{D}) \wedge (\mathbb{V},\mathbb{E}))^c = (\mathbb{P},\mathbb{D}\times\mathbb{E})^c = (\mathbb{P}^c,\mathbb{D}\times\mathbb{E})$.

That is $\forall(d,e) \in (\mathbb{D}\times\mathbb{E})$,

(i.e.) , $\mathbb{P}^c(d,e) = \{(k_m,$

$$\left\langle\begin{array}{l} \max\left\{\Psi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}(d)}}(k_m)\right)}, \Psi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}(e)}}(k_m)\right)}\right\} \\ \min\left\{\Phi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}(d)}}(k_m)\right)}, \Phi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}(e)}}(k_m)\right)}\right\} \end{array}\right\rangle$$

$$\left\langle\begin{array}{l} \max\left\{\eta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\eta^m_{\mathbb{X}(d)}}\right)}, \eta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\eta^m_{\mathbb{V}(e)}}\right)}\right\} \\ \min\left\{\zeta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}(d)}}\right)}, \zeta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}(e)}}\right)}\right\} \end{array}\right\rangle): k_m \in \mathbb{K}\}. \quad (5)$$

Take $(\mathbb{X},\mathbb{D})^c \vee (\mathbb{V},\mathbb{E})^c = (\mathbb{X}^c,\mathbb{D}) \vee (\mathbb{V}^c,\mathbb{E}) = (\mathbb{N},\mathbb{D}\times\mathbb{E})$,
where: $(\mathbb{N},(d,e)) = (\mathbb{X}^c,\mathbb{D}) \cup (\mathbb{V}^c,\mathbb{E})$
$\forall(\mathrm{d},\mathrm{e}) \in (\mathbb{D}\times\mathbb{E})$,

(i.e.), $\mathbb{N}(d,e) = \{(k_m,$

$$\left\langle\begin{array}{l} \max\left\{\Phi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}^c(d)}}(k_m)\right)}, \Phi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}^c(e)}}(k_m)\right)}\right\} \\ \min\left\{\Psi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}^c(d)}}(k_m)\right)}, \Psi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}^c(e)}}(k_m)\right)}\right\} \end{array}\right\rangle$$

$$\left\langle \begin{matrix} max\left\{ \zeta^m_{\mathbb{X}^c(d)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}^c(d)}}\right)}, \zeta^m_{\mathbb{V}^c(e)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}^c(e)}}\right)} \right\} \\ min\left\{ \eta^m_{\mathbb{X}^c(d)} e^{i2\pi\left(w_{\eta^m_{\mathbb{X}^c(d)}}\right)}, \eta^m_{\mathbb{V}^c(e)} e^{i2\pi\left(w_{\eta^m_{\mathbb{V}^c(e)}}\right)} \right\} \end{matrix} \right\rangle): k_m \in \mathbb{K}\} \quad (6)$$

$$= \{(k_m,$$

$$\left\langle \begin{matrix} \max\left\{ \Psi_{\mathbb{X}(d)}(k_m) e^{i2\pi\left(w_{\Psi_{\mathbb{X}(d)}}(k_m)\right)}, \Psi_{\mathbb{V}(e)}(k_m) e^{i2\pi\left(w_{\Psi_{V(e)}}(k_m)\right)} \right\} \\ \min\left\{ \Phi_{\mathbb{X}(d)}(k_m) e^{i2\pi\left(w_{\Phi_{\mathbb{X}(d)}}(k_m)\right)}, \Phi_{\mathbb{V}(e)}(k_m) e^{i2\pi\left(w_{\Phi_{V(e)}}(k_m)\right)} \right\} \end{matrix} \right\rangle$$

$$\left\langle \begin{matrix} max\left\{ \eta^m_{\mathbb{X}(d)} e^{i2\pi\left(w_{\eta^m_{\mathbb{X}(d)}}\right)}, \eta^m_{\mathbb{V}(e)} e^{i2\pi\left(w_{\eta^m_{\mathbb{V}(e)}}\right)} \right\} \\ min\left\{ \zeta^m_{\mathbb{X}(d)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}(d)}}\right)}, \zeta^m_{\mathbb{V}(e)} e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}(e)}}\right)} \right\} \end{matrix} \right\rangle): k_m \in \mathbb{K}\}. \quad (7)$$

It is apparent from (5) and (7) that $((\mathbb{X},\mathbb{D}) \wedge (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \vee (\mathbb{V},\mathbb{E})^c$. Comparably it can be demonstrated that $((\mathbb{X},\mathbb{D}) \vee (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \wedge (\mathbb{V},\mathbb{E})^c$.

Definition 3.7. Suppose $(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ be two $\mathbb{CLDFSS}$. If $\mathbb{G} = \mathbb{D} \cup \mathbb{E}$ and for all $\varkappa \in \mathbb{G}$, then the Extended Union $(\mathbb{L},\mathbb{G})$ of $(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ is designated by:

$$\mathbb{L}(\varkappa) = \begin{cases} \mathbb{X}(\varkappa) & \text{if } \varkappa \in \mathbb{D} \setminus \mathbb{E} \\ \mathbb{V}(\varkappa) & \text{if } \varkappa \in \mathbb{E} \setminus \mathbb{D} \\ \mathbb{X}(\varkappa) \cup \mathbb{V}(\varkappa) & \text{if } \varkappa \in \mathbb{D} \cap \mathbb{E} \end{cases}$$

(i.e.), $\forall \boldsymbol{\varkappa} \in \mathbb{D} \cap \mathbb{E}$,

$$\Rightarrow \mathbb{L}(\boldsymbol{\varkappa}) = \{(\boldsymbol{k_m},$$

$$\langle \boldsymbol{\Phi}_{\mathbb{L}(\varkappa)}(\boldsymbol{k_m}) \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Phi_{\mathbb{I}(\varkappa)}}(k_m)\right)}, \boldsymbol{\Psi}_{\mathbb{L}(\varkappa)}(\boldsymbol{k_m}) \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Psi_{\mathbb{L}(\varkappa)}}(k_m)\right)} \rangle$$

$$\langle \boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{L}(\varkappa)} \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w^{m}_{\zeta_{\mathbb{L}}}\right)}, \boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{L}(\varkappa)} \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w^{m}_{\eta_{\mathbb{L}(\varkappa)}}\right)} \rangle : \boldsymbol{k_m} \in \mathbb{K}\}, \quad (8)$$

where:

$$\left\{ \begin{matrix} \Phi_{\mathbb{L}(\varkappa)}(k_m) = \max\{\Phi_{\mathbb{X}(\varkappa)}(k_m), \Phi_{\mathbb{V}(\varkappa)}(k_m)\} \\ w_{\Phi_{\mathbb{L}(\varkappa)}}(k_m) = \max\left\{w_{\Phi_{\mathbb{X}(\varkappa)}}(k_m), w_{\Phi_{\mathbb{V}(\varkappa)}}(k_m)\right\} \\ \Psi_{\mathbb{L}(\varkappa)}(k_m) = \min\{\Psi_{\mathbb{X}(\varkappa)}(k_m), \Psi_{\mathbb{V}(\varkappa)}(k_m)\} \\ w_{\Psi_{\mathbb{L}(\varkappa)}}(k_m) = \min\left\{w_{\Psi_{\mathbb{X}(\varkappa\varkappa)}}(k_m), w_{\Psi_{\mathbb{V}(\varkappa)}}(k_m)\right\} \\ \zeta^m_{\mathbb{L}(\varkappa)} = \max\{\zeta^m_{\mathbb{X}(\varkappa)}, \zeta^m_{\mathbb{V}(\varkappa)}\} \\ w^m_{S^m_{\mathbb{L}(\varkappa)}} = \max\left\{w_{\zeta^m_{\mathbb{X}(\varkappa)}}, w_{\zeta^m_{\mathbb{V}(\varkappa)}}\right\} \\ \eta^m_{\mathbb{L}(\varkappa)} = \min\{\eta^m_{\mathbb{X}(\varkappa\varkappa)}, \eta^m_{\mathbb{V}(\varkappa)}\} \\ w^m_{\eta^m_{\mathbb{L}(\varkappa)}} = \min\left\{w^m_{\eta^m_{\mathbb{X}(\varkappa)}}, w^m_{\eta^m_{\mathbb{V}(\varkappa)}}\right\} \end{matrix} \right\}_{\{m=1,2,\dots,s\}}.$$

Definition 3.8. Suppose $(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ be two $\mathbb{CLDFSS}$. If $\mathbb{G} = \mathbb{D} \cup \mathbb{E}$ and for all $\varkappa \in \mathbb{G}$, then the Extended Intersection $(\mathbb{H},\mathbb{G})$ of $(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ is specified as:

$$\mathbb{H}(\boldsymbol{\varkappa}) = \begin{cases} \mathbb{X}(\boldsymbol{\varkappa}) & \text{if } \boldsymbol{\varkappa} \in \mathbb{D} \setminus \mathbb{E} \\ \mathbb{V}(\boldsymbol{\varkappa}) & \text{if } \boldsymbol{\varkappa} \in \mathbb{E} \setminus \mathbb{D} \\ \mathbb{X}(\boldsymbol{\varkappa}) \cap \mathbb{V}(\boldsymbol{\varkappa}) & \text{if } \boldsymbol{\varkappa} \in \mathbb{D} \cap \mathbb{E} \end{cases},$$

(i.e.), $\forall \boldsymbol{\varkappa} \in \mathbb{D} \cap \mathbb{E}$,

$$\Rightarrow \mathbb{H}(\boldsymbol{\varkappa}) = \{(\boldsymbol{k_m},$$

$$\langle \boldsymbol{\Phi}_{\mathbb{H}(\varkappa)}(\boldsymbol{k_m}) \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Phi_{\mathbb{H}}}(k_m)\right)}, \boldsymbol{\Psi}_{\mathbb{H}(\varkappa)}(\boldsymbol{k_m}) \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\Psi_{\mathbb{H}(\varkappa)}}(k_m)\right)} \rangle$$

$$\langle \boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)} \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w^{m}_{\zeta_{\mathbb{H}}}(\varkappa)\right)}, \boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)} \boldsymbol{e}^{\boldsymbol{i2\pi}\left(w_{\eta^m_{\mathbb{H}(\varkappa)}}\right)} \rangle): \boldsymbol{k_m} \in \mathbb{K}\}, \quad (9)$$

where:

$$\left\{ \begin{matrix} \boldsymbol{\Phi}_{\mathbb{H}(\varkappa)}(\boldsymbol{k_m}) = \mathbf{min}\{\boldsymbol{\Phi}_{\mathbb{X}(\varkappa)}(\boldsymbol{k_m}), \boldsymbol{\Phi}_{\mathbb{V}(\varkappa)}(\boldsymbol{k_m})\} \\ \boldsymbol{w}_{\boldsymbol{\Phi}_{\mathbb{H}(\varkappa)}}(\boldsymbol{k_m}) = \mathbf{min}\left\{\boldsymbol{w}_{\boldsymbol{\Phi}_{\mathbb{X}(\varkappa)}}(\boldsymbol{k_m}), \boldsymbol{w}_{\boldsymbol{\Phi}_{\mathbb{V}(\varkappa)}}(\boldsymbol{k_m})\right\} \\ \boldsymbol{\Psi}_{\mathbb{H}(\varkappa)}(\boldsymbol{k_m}) = \mathbf{max}\{\boldsymbol{\Psi}_{\mathbb{X}(\varkappa)}(\boldsymbol{k_m}), \boldsymbol{\Psi}_{\mathbb{V}(\varkappa)}(\boldsymbol{k_m})\} \\ \boldsymbol{w}_{\boldsymbol{\Psi}_{\mathbb{H}(\varkappa)}}(\boldsymbol{k_m}) = \mathbf{max}\left\{\boldsymbol{w}_{\boldsymbol{\Psi}_{\mathbb{X}(\varkappa\varkappa)}}(\boldsymbol{k_m}), \boldsymbol{w}_{\boldsymbol{\Psi}_{\mathbb{V}(\varkappa)}}(\boldsymbol{k_m})\right\} \\ \boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)} = \mathbf{min}\{\boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{X}(\varkappa)}, \boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{V}(\varkappa)}\} \\ \boldsymbol{w}^{\boldsymbol{m}}_{\boldsymbol{S}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)}} = \mathbf{min}\left\{\boldsymbol{w}_{\boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{X}(\varkappa)}}, \boldsymbol{w}_{\boldsymbol{\zeta}^{\boldsymbol{m}}_{\mathbb{V}(\varkappa)}}\right\} \\ \boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)} = \mathbf{max}\{\boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{X}(\varkappa\varkappa)}, \boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{V}(\varkappa)}\} \\ \boldsymbol{w}^{\boldsymbol{m}}_{\boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{H}(\varkappa)}} = \mathbf{max}\left\{\boldsymbol{w}^{\boldsymbol{m}}_{\boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{X}(\varkappa)}}, \boldsymbol{w}^{\boldsymbol{m}}_{\boldsymbol{\eta}^{\boldsymbol{m}}_{\mathbb{V}(\varkappa)}}\right\} \end{matrix} \right\}_{\{\boldsymbol{m}=\mathbf{1},\mathbf{2},\dots,\boldsymbol{s}\}}.$$

Theorem 3.9. Suppose that $(\mathbb{X},\mathbb{D}),(\mathbb{V},\mathbb{E})$ and $(\mathbb{W},\mathbb{O})$ depicts three $\mathbb{CLDFSS}$. Then the subsequent properties hold.

1. $(\mathbb{X},\mathbb{D}) \cup (\mathbb{X},\mathbb{D}) = (\mathbb{X},\mathbb{D})$
2. $(\mathbb{X},\mathbb{D}) \cap (\mathbb{X},\mathbb{D}) = (\mathbb{X},\mathbb{D})$
3. $(\mathbb{X},\mathbb{D}) \cup (\mathbb{V},\mathbb{E}) = (\mathbb{V},\mathbb{E}) \cup (\mathbb{X},\mathbb{D})$
4. $(\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E}) = (\mathbb{V},\mathbb{E}) \cap (\mathbb{X},\mathbb{D})$
5. $((\mathbb{X},\mathbb{D}) \cup (\mathbb{V},\mathbb{E})) \cup (\mathbb{W},\mathbb{D}) = (\mathbb{X},\mathbb{D}) \cup ((\mathbb{V},\mathbb{E}) \cup (\mathbb{W},\mathbb{O}))$
6. $((\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E})) \cap (\mathbb{W},\mathbb{D}) = (\mathbb{X},\mathbb{D}) \cap ((\mathbb{V},\mathbb{E}) \cap (\mathbb{W},\mathbb{O}))$

Proof. The proof is apparent.

Theorem 3.10. Suppose that $(\mathbb{X},\mathbb{D})$ and $(\mathbb{V},\mathbb{E})$ depicts two $\mathbb{CLDFSS}$, then:

1. $((\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \cup (\mathbb{V},\mathbb{E})^c$
2. $((\mathbb{X},\mathbb{D}) \cup (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \cap (\mathbb{V},\mathbb{E})^c$

Proof. Let $(\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E}) = (\mathbb{H},\mathbb{G})$, where $\mathbb{G} = \mathbb{D} \cup \mathbb{E}$ and $\varkappa \in \mathbb{G}$,

$$\mathbb{H}(\varkappa) = \begin{cases} \mathbb{X}(\varkappa) & \text{if } \varkappa \in \mathbb{D} \setminus \mathbb{E} \\ \mathbb{V}(\varkappa) & \text{if } \varkappa \in \mathbb{E} \setminus \mathbb{D} \\ \mathbb{X}(\varkappa) \cap \mathbb{V}(\varkappa) & \text{if } \varkappa \in \mathbb{D} \cap \mathbb{E} \end{cases}$$

(i.e.), $\forall \varkappa \in \mathbb{D} \cap \mathbb{E}$,

$$\Rightarrow \mathbb{H}(\varkappa) = \{(k_m,$$

$$\left\langle \begin{matrix} \min\left\{\Phi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}(d)}(k_m)}\right)}, \Phi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}(e)}(k_m)}\right)}\right\} \\ \max\left\{\Psi_{\mathbb{X}(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}(d)}(k_m)}\right)}, \Psi_{\mathbb{V}(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}(e)}(k_m)}\right)}\right\} \end{matrix} \right\rangle$$

$$\left\langle \begin{matrix} \min\left\{\zeta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}(d)}}\right)}, \zeta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}(e)}}\right)}\right\} \\ \max\left\{\eta^m_{\mathbb{X}(d)}e^{i2\pi\left(w_{\eta^m_{\mathbb{X}(d)}}\right)}, \eta^m_{\mathbb{V}(e)}e^{i2\pi\left(w_{\eta^m_{\mathbb{V}(e)}}\right)}\right\} \end{matrix} \right\rangle): k_m \in \mathbb{K}\}, \quad (10)$$

so that, $((\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E}))^c = (\mathbb{H},\mathbb{G})^c$ and $\mathbb{H}^c(\varkappa) = (\mathbb{H}(\varkappa))^c$,

$$\text{Then } (\mathbb{H}(\varkappa))^c = \begin{cases} (\mathbb{X}(\varkappa))^c & \text{if } \varkappa \in \mathbb{D} \setminus \mathbb{E} \\ (\mathbb{V}(\varkappa))^c & \text{if } \varkappa \in \mathbb{E} \setminus \mathbb{D} \\ (\mathbb{X}(\varkappa) \cap \mathbb{V}(\varkappa))^c & \text{if } \varkappa \in \mathbb{D} \cap \mathbb{E} \end{cases}$$

(i.e.), $\forall \varkappa \in \mathbb{D} \cap \mathbb{E}$,

$\Rightarrow (\mathbb{X}(\varkappa) \cap \mathbb{V}(\varkappa))^c = \{(k_m,$

$$\left\langle \begin{matrix} \max\left\{\Psi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}^c(d)}(k_m)}\right)}, \Psi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}^c(e)}(k_m)}\right)}\right\} \\ \min\left\{\Phi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}^c(d)}(k_m)}\right)}, \Phi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}^c(e)}(k_m)}\right)}\right\} \end{matrix} \right\rangle$$

$$\left\langle \begin{matrix} \max\left\{\eta^m_{\mathbb{X}^c(d)}e^{i2\pi\left(w_{\eta^m_{\mathbb{X}^c(d)}}\right)}, \eta^m_{\mathbb{V}^c(e)}e^{i2\pi\left(w_{\eta^m_{\mathbb{V}^c(e)}}\right)}\right\} \\ \min\left\{\zeta^m_{\mathbb{X}^c(d)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}^c(d)}}\right)}, \zeta^m_{\mathbb{V}^c(e)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}^c(e)}}\right)}\right\} \end{matrix} \right\rangle): k_m \in \mathbb{K}\} \quad (11)$$

Now $(\mathbb{X},\mathbb{D})^c = (\mathbb{X}^c,\mathbb{D})$ and $(\mathbb{V},\mathbb{E})^c = (\mathbb{V}^c,\mathbb{E})$, so that $(\mathbb{X},\mathbb{D})^c \cup (\mathbb{V},\mathbb{E})^c = (\mathbb{X}^c,\mathbb{D}) \cup (\mathbb{V}^c,\mathbb{E})$.

Take $(\mathbb{X}^c,\mathbb{D}) \cup (\mathbb{V}^c,\mathbb{E}) = (\mathbb{J}^c,\mathbb{G})$, where $\mathbb{G} = \mathbb{D} \cup \mathbb{E}$ and

$$\text{Then } (\mathbb{J}^c(\varkappa)) = \begin{cases} (\mathbb{X}^c(\varkappa)) & \text{if } \varkappa \in \mathbb{D} \setminus \mathbb{E} \\ (\mathbb{V}^c(\varkappa)) & \text{if } \varkappa \in \mathbb{E} \setminus \mathbb{D} \\ \mathbb{X}^c(\varkappa) \cup \mathbb{V}^c(\varkappa) & \text{if } \varkappa \in \mathbb{D} \cap \mathbb{E} \end{cases}$$

(i.e.), $\forall \varkappa \in \mathbb{G}$

$\Rightarrow \mathbb{X}^c(\varkappa) \cup \mathbb{V}^c(\varkappa) = \{(k_m,$

$$\left\langle \begin{matrix} \max\left\{\Psi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{X}^c(d)}(k_m)}\right)}, \Psi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Psi_{\mathbb{V}^c(e)}(k_m)}\right)}\right\} \\ \min\left\{\Phi_{\mathbb{X}^c(d)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{X}^c(d)}(k_m)}\right)}, \Phi_{\mathbb{V}^c(e)}(k_m)e^{i2\pi\left(w_{\Phi_{\mathbb{V}^c(e)}(k_m)}\right)}\right\} \end{matrix} \right\rangle$$

$$\left\langle \begin{matrix} \max\left\{\eta^m_{\mathbb{X}^c(d)}e^{i2\pi\left(w_{\eta^m_{\mathbb{X}^c(d)}}\right)}, \eta^m_{\mathbb{V}^c(e)}e^{i2\pi\left(w_{\eta^m_{\mathbb{V}^c(e)}}\right)}\right\} \\ \min\left\{\zeta^m_{\mathbb{X}^c(d)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{X}^c(d)}}\right)}, \zeta^m_{\mathbb{V}^c(e)}e^{i2\pi\left(w_{\zeta^m_{\mathbb{V}^c(e)}}\right)}\right\} \end{matrix} \right\rangle): k_m \in \mathbb{K}\} \quad (12)$$

It is apparent from (11) and (12) that $((\mathbb{X},\mathbb{D}) \cap (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \cup (\mathbb{V},\mathbb{E})^c$. Comparably it can be demonstrated that $((\mathbb{X},\mathbb{D}) \cup (\mathbb{V},\mathbb{E}))^c = (\mathbb{X},\mathbb{D})^c \cap (\mathbb{V},\mathbb{E})^c$.

4. A MATHEMATICAL SIMULATION OF $\mathbb{CLDFSS}$

Furtherance of $\mathbb{MCDM}$ model for $\mathbb{CLDFSS}$ has been interpreted with a numerical specimen to access the real-life situation using mathematical modeling. Some of the required essential definitions are particularized as beneath.

Definition 4.1. A square table with an identical number of rows and columns is called a comparison table. Both rows and columns are specified with the elements of a universal set $\boldsymbol{k_1, k_2, \ldots, k_s}$ and the entries are $\mathbb{Z}_{\boldsymbol{tu}}, \boldsymbol{t}, \boldsymbol{u} = \mathbf{1}$, 2, ...s characterized by $\mathbb{Z}_{\boldsymbol{tu}}$ = number of attributes for which the score of $\boldsymbol{k_t}$ is excess or equal to the score of $\boldsymbol{k_u}$.

Evidently, $0 \le \mathbb{Z}_{tu} \le y$ and $\mathbb{Z}_{tt} = y$ where, y symbolizes the number of attributes in $\mathbb{CLDFSS}$ (i.e., the number of attributes for which the score of k_t is excess or equal to the score of k_t itself).

Definition 4.2. A Row-Sum of k_t is specified by $\mathbb{R}_t$ and it is computed by employing the mathematical equation:

$$\mathbb{R}_t = \sum_{u=1}^{s} \mathbb{Z}_{tu}. \quad (13)$$

Definition 4.3. A Column-Sum of k_u is specified by $\mathbb{C}_u$ and it is computed by employing the mathematical equation:

$$\mathbb{C}_u = \sum_{t=1}^{s} \mathbb{Z}_{tu}. \quad (14)$$

Definition 4.4. A Score of k_t is specified by $\mathbb{S}_t$ and characterized as:

$$\mathbb{S}_t = \mathbb{R}_t - \mathbb{C}_t. \quad (15)$$

4.1. A Unique Paradigm Centered Around $\mathbb{MCDM}$ With Application

In this module, a unique paradigm based on $\mathbb{MCDM}$ model of $\mathbb{CLDFSS}$ in accordance with the intent of developing a credible approach to determine a pertinent antenna in a communication device. Innovative mathematical modeling has been specified and data are collected under $\mathbb{CLDFSS}$ environment.

4.1.1 Mathematical Modeling

In this segment, an algorithm is intricately developed to address real-life problems using mathematical modeling.

ALGORITHM

$\mathbb{INPUT}$:

$\mathbb{STEP}$ **1:** Originate the $\mathbb{CLDFSS}(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$.

$\mathbb{CALCULATIONS}$:

$\mathbb{STEP}$ **2:** Quantify the Resultant- $\mathbb{CLDFSS}$ $\left(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D}\right)$ from $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$ by using Equation (2).

$\mathbb{STEP}$ **3:** Evaluate the Accuracy function using Equation (1).

$\mathbb{STEP}$ **4:** Fabricate the Comparison table of $\left(\mathbb{X}_{\mathbb{Q}}, \mathbb{D}\right)$.

$\mathbb{STEP}$ **5:** Enumerate the value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and Determine the Score value $\mathbb{S}_t \forall t$ by using the Equations (13), (14), (15) respectively.

$\mathbb{OUTPUT}$:

$\mathbb{STEP}$ **6:** Establish the Hierarchy between the alternatives based on the Score value.

$\mathbb{FINAL}$ $\mathbb{DECISION}$:

$\mathbb{STEP}$ **7:** A final decision will be contingent upon the alternative in the core.

The Schema chart of the aforementioned algorithm is characterized in Fig. 1.

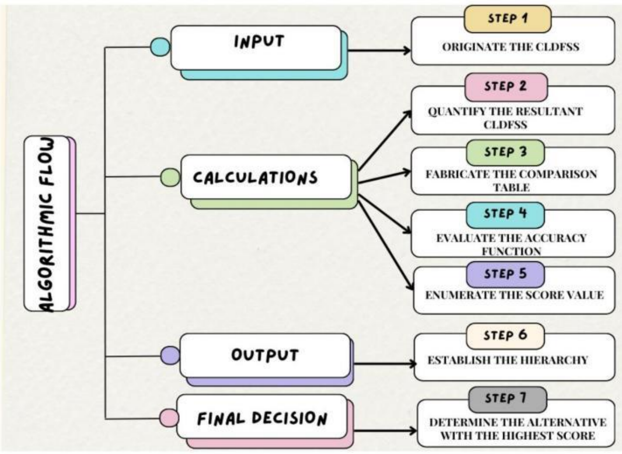

Fig. 1. Schema Chart

4.2. Interpretative Example

The primary significance of the existence of the armed forces is to safeguard a country from external attacks. Also, the military accompanies the police in resolving any internal security problem. The military can help the country yield some revenue when they depart for peacekeeping operations across the world. The military undertakes humanitarian services during catastrophic events. The military is the first squadron of people called upon during catastrophic events. They assist catastrophe victims and perform rescue operations during calamities.

In intending to perform all these operations, one of the major factors involved is communication between the armed forces. The effective and explicit exchange of information has consistently been paramount to every successful military operation. This is especially true in modern times. Enriching the communication facility between the armed forces is as equally important as ruining the communication between the enemy forces.

Better communication has the potential to retain a country's safety. Antennas play a vital role in military communication devices. The antenna is a primary device that is comprised of a series of conductors. On the receiver's end, an antenna adopts electromagnetic radiation and transforms it into radio frequency electrical impulses. The antenna keeps the ground unit safe and also helps in uninterrupted communication. The advantages of effective communication between armed forces are listed in the form of a histogram, which is specified in Fig.2.

Fig. 2. Advantages of effective communication between the armed force

A structure that positively influences the communication device is the antenna. The appropriate selection of antennas in communication devices is important. Supposing that $\mathbb{K} = \{k_1, k_2, k_3\}$ is the set of possible of antennas and let $\mathbb{D} = \{d_1, d_2, d_3\}$ be the attribute set, whereas $d_1 =$ Performance, $d_2 =$ Technical Support and $d_3 =$ Safety Measure. A year has four seasons and the attributes have distinct values for each season. So, the intention is to present an innovative mathematical model for selecting pertinent antennae in communication devices based on the verdict of two experts, namely $\{\mathbb{F}, \mathbb{I}\}$.

The categorizing of attributes is particularized as follows.

1. The attribute "Performance" directs that the alternative is "high" or "low".
2. The attribute "Technical Support" directs that the alternative is "more" or "less".
3. The attribute "Safety Measure" directs that the alternative is "highly required" or "lowly required".

A depiction in tabular form is shown in Table 6.

The approximation "Performance" has estimates $\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle,\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{1}{4}\right)\right\rangle$ in accordance with the decision made by the expert $\mathbb{F}$ for the alternative k_1. It demonstrates that k_1 has a 0.7 truth score in the summer season and a 0.3 false score in the autumn season. The reference parameter score $\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{1}{4}\right)\right\rangle$ signifies that k_1 should be 0.7 score high in the summer season and should be 0.3 score low in the spring season. All other data are provided similarly.

Table 6. Categorizing of attributes

Attributes	Characteristics of $\mathbb{CLDFSS}$
"Performance"	$(\langle\mathbb{CVMS},\mathbb{CVNMS}\rangle,$ $\langle$ high,low $\rangle)$
"Technical Support"	$(\langle\mathbb{CVMS},\mathbb{CVNMS}\rangle,$ $\langle$ more, less $\rangle)$
"Safety Measure"	$(\langle\mathbb{CVMS},\mathbb{CVNMS}\rangle,$ $\langle$ high, low $\rangle)$

4.3. A Systematic Algorithmic Process

$\mathbb{STEP}$ 1: Originate the $\mathbb{CLDFSS}(\mathbb{X}_{\mathbb{F}},\mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}},\mathbb{D})$ given in Tables 7 and 8.

Table 7. $\mathbb{CLDFSS}(\mathbb{X}_{\mathbb{F}},\mathbb{D})$

$(\mathbb{X}_{\mathbb{F}},\mathbb{D})$	d_1	d_2	d_3
k_1	$\left\langle\left(0.7,\frac{4}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle,$	$\left\langle\left(0.6,\frac{3}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.6,\frac{4}{4}\right),\left(0.4,\frac{3}{4}\right)\right\rangle,$
	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{1}{4}\right)\right\rangle$
k_2	$\left\langle\left(0.6,\frac{3}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.7,\frac{1}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle,$	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$
	$\left\langle\left(0.7,\frac{1}{4}\right),\left(0.3,\frac{2}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{1}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{3}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$
k_3	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle,$	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.7,\frac{3}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$
	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.7,\frac{1}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{3}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$

Table 8. $\mathbb{CLDFSS}(\mathbb{X}_{\mathbb{I}},\mathbb{D})$

$(\mathbb{X}_{\mathbb{I}},\mathbb{D})$	d_1	d_2	d_3
k_1	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle,$	$\left\langle\left(0.5,\frac{2}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.6,\frac{3}{4}\right),\left(0.4,\frac{4}{4}\right)\right\rangle,$
	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.6,\frac{1}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{3}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$
k_2	$\left\langle\left(0.7,\frac{2}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.8,\frac{3}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle,$
	$\left\langle\left(0.8,\frac{1}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle$	$\left\langle\left(0.9,\frac{3}{4}\right),\left(0.1,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$
k_3	$\left\langle\left(0.6,\frac{3}{4}\right),\left(0.3,\frac{4}{4}\right)\right\rangle,$	$\left\langle\left(0.7,\frac{4}{4}\right),\left(0.2,\frac{3}{4}\right)\right\rangle,$	$\left\langle\left(0.7,\frac{4}{4}\right),\left(0.3,\frac{3}{4}\right)\right\rangle,$
	$\left\langle\left(0.9,\frac{2}{4}\right),\left(0.1,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.1,\frac{1}{4}\right)\right\rangle$	$\left\langle\left(0.8,\frac{2}{4}\right),\left(0.2,\frac{1}{4}\right)\right\rangle$

$\mathbb{STEP}$ 2**:** Quantify the Resultant- $\mathbb{CLDFSS}$ $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$ from $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$ by using Definition 3.2. which is specified in Table 9.

Table 9. Resultant- $\mathbb{CLDFSS}$ $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$

	$d_1 \times d_1$	$d_1 \times d_2$	$d_1 \times d_3$	$d_2 \times d_1$	$d_2 \times d_2$
k_1	$(0.7,\frac{2}{4}),(0.3,\frac{3}{4})$ $(0.8,\frac{2}{4}),(0.3,\frac{1}{4})$	$(0.5,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.6,\frac{1}{4}),(0.3,\frac{3}{4})$	$(0.6,\frac{3}{4}),(0.4,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.6,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.7,\frac{2}{4}),(0.3,\frac{1}{4})$	$(0.5,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.6,\frac{1}{4}),(0.3,\frac{3}{4})$
k_2	$(0.6,\frac{2}{4}),(0.3,\frac{3}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$	$(0.6,\frac{3}{4}),(0.3,\frac{4}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{2}{4})$	$(0.6,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{2}{4})$	$(0.7,\frac{1}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{1}{4}),(0.2,\frac{4}{4})$	$(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$ $(0.8,\frac{1}{4}),(0.2,\frac{3}{4})$
k_3	$(0.6,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.7,\frac{2}{4}),(0.2,\frac{3}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.7,\frac{2}{4}),(0.3,\frac{3}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.6,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$	$(0.7,\frac{2}{4}),(0.2,\frac{4}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$
	$d_2 \times d_3$	$d_3 \times d_1$	$d_3 \times d_2$	$d_3 \times d_3$	-
k_1	$(0.6,\frac{3}{4}),(0.4,\frac{4}{4})$ $(0.7,\frac{2}{4}),(0.3,\frac{1}{4})$	$(0.6,\frac{2}{4}),(0.4,\frac{3}{4})$ $(0.7,\frac{2}{4}),(0.3,\frac{1}{4})$	$(0.5,\frac{2}{4}),(0.4,\frac{2}{4})$ $(0.6,\frac{1}{4}),(0.3,\frac{3}{4})$	$(0.6,\frac{3}{4}),(0.4,\frac{4}{4})$ $(0.7,\frac{2}{4}),(0.3,\frac{1}{4})$	-
k_2	$(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$ $(0.8,\frac{1}{4}),(0.2,\frac{3}{4})$	$(0.7,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{1}{4}),(0.2,\frac{3}{4})$	$(0.7,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{3}{4}),(0.2,\frac{1}{4})$	$(0.7,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	-
k_3	$(0.7,\frac{2}{4}),(0.3,\frac{4}{4})$ $(0.7,\frac{1}{4}),(0.3,\frac{3}{4})$	$(0.6,\frac{3}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.7,\frac{3}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	$(0.7,\frac{3}{4}),(0.3,\frac{4}{4})$ $(0.8,\frac{2}{4}),(0.2,\frac{1}{4})$	-

$\mathbb{STEP}$ 3**:** Evaluate the Accuracy function using Definition 2.7. which is particularized in Table 10.

Table 10. Table value of Accuracy function

	$d_1 \times d_1$	$d_1 \times d_2$	$d_1 \times d_3$	$d_2 \times d_1$	$d_2 \times d_2$	$d_2 \times d_3$	$d_3 \times d_1$	$d_3 \times d_2$	$d_3 \times d_3$
k_1	0.74375	0.7625	0.78125	0.625	0.65	0.78125	0.71875	0.7125	0.78125
k_2	0.76875	0.76875	0.7375	0.78125	0.78125	0.75	0.7	0.8125	0.75
k_3	0.7375	0.70625	0.71875	0.8	0.8	0.8125	0.76875	0.78125	0.78125

$\mathbb{STEP}$ 4:Fabricate the Comparison table of $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D})$)(Table 11).

Table 11. Comparison table of $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D})$

	k_1	k_2	k_3
k_1	9	4	4
k_2	5	9	4
k_3	6	5	9

$\mathbb{STEP}$ 5: Enumerate the value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and Determine the Score value $\mathbb{S}_t \forall t$, which was identified in Table 12.

Table 12. Score Table

	Row-Sum ($\mathbb{R}_t$)	Column-Sum ($\mathbb{C}_t$)	Score ($\mathbb{S}_t$)
k_1	17	20	-3
k_2	18	18	0
k_3	20	17	3

$\mathbb{STEP}$ 6: Establish the Hierarchy between the alternatives based on the Score value.

The Hierarchy between the alternatives is based on the Score ($\mathbb{S}_t$) is $k_3 > k_2 > k_1$.

$\mathbb{STEP}$ 7: A final decision will be contingent upon the alternative in the core.

The ideal grade is 3 based on the score value obtained and the verdict is in the preference of opting for k_3.

5. ASSESSMENT BY COMPARISON

The suggested methodology is the most effective way to incorporate ambiguous and uncertain facts into a situation requiring decision-making. A logical comparison of the proposed algorithm with the other existing algorithms of Roy and Maji [33] and Liu et al. [34] is given in Table 13 and Table 19 and performed to demonstrate the proposed approach's effectiveness and adaptability.

Table 13. $\mathbb{ALGORITHM}$ $\mathbb{1}$ – Roy and Maji [33]

$\mathbb{STEP}$ 1:	Originate the $\mathbb{FSS}$ $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$.
$\mathbb{STEP}$ 2:	Quantify the Resultant- $\mathbb{FSS}$ $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$ from $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$.
$\mathbb{STEP}$ 3:	Fabricate the Comparison table of $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$.
$\mathbb{STEP}$ 4:	Enumerate the value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and Determine the Score value $\mathbb{S}_t \forall t$.
$\mathbb{STEP}$ 5 :	Establish the Hierarchy between the alternatives based on the Score value.

5.1. A Systematic Algorithmic Process

$\mathbb{STEP}$ 1: The $\mathbb{FSS}$s$(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$ are given in Tables 14 and 15.

Table 14. $\mathbb{FSS}(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$

$(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$	d_1	d_2	d_3
k_1	0.7	0.6	0.6
k_2	0.6	0.6	0.7
k_3	0.7	0.8	0.7

Table 15. $\mathbb{FSS}(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$

$(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$	d_1	d_2	d_3
k_1	0.7	0.5	0.6
k_2	0.7	0.8	0.8
k_3	0.8	0.8	0.8

$\mathbb{STEP}$ 2: The Resultant- $\mathbb{FSS}(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$ from $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ and $(\mathbb{X}_{\mathbb{I}}, \mathbb{D})$ is specified in Table 16.

Table 16. Resultant- $\mathbb{FSS}(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$

	$d_1 \times d_1$	$d_1 \times d_2$	$d_1 \times d_3$	$d_2 \times d_1$	$d_2 \times d_2$	$d_2 \times d_3$	$d_3 \times d_1$	$d_3 \times d_2$	$d_3 \times d_3$
k_1	0.7	0.5	0.6	0.6	0.5	0.6	0.6	0.5	0.6
k_2	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7
k_3	0.7	0.7	0.7	0.8	0.8	0.8	0.7	0.7	0.7

$\mathbb{STEP}$ 3: The Comparison table of $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$ is designated in Table 17.

Table 17. Comparison table of $(\mathbb{X}_{\mathbb{Q}}, \mathbb{D} \times \mathbb{D})$

	k_1	k_2	k_3
k_1	9	4	1
k_2	8	9	3
k_3	9	9	9

$\mathbb{STEP}$ 4: The value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and the Score value $\mathbb{S}_t \forall t$ are identified in Table 18.

Table 18. Score Table

	Row-Sum ($\mathbb{R}_t$)	Column-Sum ($\mathbb{C}_t$)	Score ($\mathbb{S}_t$)
k_1	14	26	-12
k_2	20	22	-2
k_3	27	13	14

$\mathbb{STEP}$ 5: Establish the Hierarchy between the alternatives based on the Score value. The Hierarchy between the alternatives is based on the Score ($\mathbb{S}_t$) is $k_3 > k_2 > k_1$.

Table 19. $\mathbb{ALGORITHM}$ 2 - Liu et al. [34]

$\mathbb{STEP}$ 1:	Originate the $\mathbb{FSS}(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$.
$\mathbb{STEP}$ 2:	Fabricate the Comparison table of $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$.
$\mathbb{STEP}$ 3:	Enumerate the value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and Determine the Score value $\mathbb{S}_t \forall t$.
$\mathbb{STEP}$ 4:	Establish the Hierarchy between the alternatives based on the Score value.

5.2 A Systematic Algorithmic Process

$\mathbb{STEP}$ 1: The $\mathbb{FSS}(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ is given in Table 20.

Table 20. $\mathbb{FSS}(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$

$(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$	d_1	d_2	d_3
k_1	0.7	0.6	0.6
k_2	0.6	0.6	0.7
k_3	0.7	0.8	0.7

$\mathbb{STEP}$ 2: The Comparison table of $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$ is designated in Table 21.

Table 21. Comparison table of $(\mathbb{X}_{\mathbb{F}}, \mathbb{D})$

	k_1	k_2	k_3
k_1	3	2	1
k_2	2	3	1
k_3	3	3	3

$\mathbb{STEP}$ 3: The value of $\mathbb{R}_t$ and $\mathbb{C}_t \forall t$ and the Score value $\mathbb{S}_t \forall t$ are identified in Table 22.

Table 22. Score Table

	Row-Sum ($\mathbb{R}_t$)	Column-Sum ($\mathbb{C}_t$)	Score ($\mathbb{S}_t$)
k_1	6	8	-2
k_2	6	8	-2
k_3	9	5	4

$\mathbb{STEP}$ 4: Establish the Hierarchy between the alternatives based on the Score value. The Hierarchy between the alternatives is based on the Score ($\mathbb{S}_t$) is $k_3 > k_2 = k_1$

In this comparison, the accuracy of the final ranking evaluation is demonstrated. The comparison table for the subsections 4.1, 5.1, and 5.2 is given in Table 23. The Bar Graph Infographic is visualized in Fig. 3.

Table 23. Assessment by Comparison

Assessment by Comparison		
Models	Ranking of the alternatives	Pertinent Antenna
Proposed $\mathbb{CLDFSS}$	$k_3 > k_2 > k_1$	k_3
Roy and Maji [33]	$k_3 > k_2 > k_1$	k_3
Liu et al. [34]	$k_3 > k_2 = k_1$	k_3

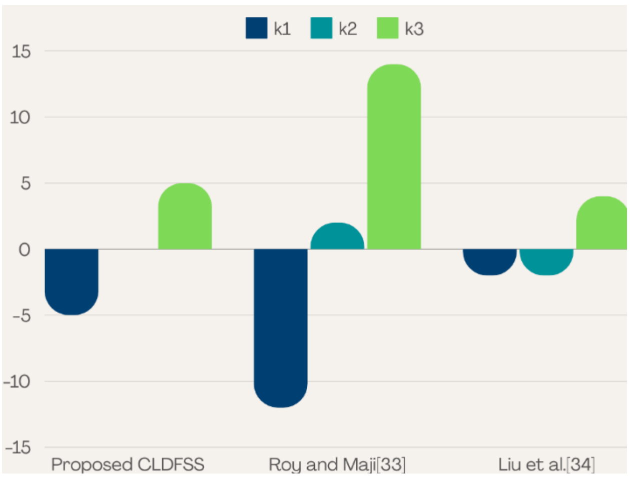

Fig. 3. Bar Graph Infographic

5.3 Advantages of the Proposed Methodology

1. In contrast to existing approaches, the $\mathbb{CLDFSS}$ framework offers a more thorough and accurate modeling of complex decision-making scenarios by integrating $\mathbb{CLDFS}$ with $\mathbb{SS}$. This theoretical improvement offers a reliable method for handling partial, ambiguous, and inconsistent information.
2. An algorithm's deployment of fundamental operations and the accuracy function within $\mathbb{CLDFSS}$ allows for an accurate assessment of available antenna alternatives. This method ensures that the chosen antennas satisfy the necessary criteria more than conventional techniques by enabling thorough performance evaluation and comparison.

5.4 Limitations and Practical Implications

Limitations

1. Implementation Complexity: The $\mathbb{CLDFSS}$ technique requires extensive computational resources and specialized knowledge, and it entails intricate calculations and modeling that may provide difficulties in real-world applications.
2. Data Dependency: The quality of the input data has a major impact on the accuracy of the $\mathbb{CLDFSS}$ method. The effectiveness of antenna selection can be affected by inaccurate or incomplete data, which can also restrict the use of the methodology in situations where the information is unreliable.

Practical Implications:

1. Enhanced Decision-Making: By offering a systematic approach with accurate evaluation tools, the suggested technique improves antenna selection in military communications and increases operational efficiency and reliability.
2. Versatile Applications: The versatility of the $\mathbb{CLDFSS}$ framework enables its application in several domains, such as emergency response systems, satellite technology, defense communications, and aerospace. This flexibility provides customized solutions that improve communication systems efficacy in a range of operational contexts.

6. CONCLUSION

The paper elaborates on the origin of the development and extension of $\mathbb{FS}$ theory by identifying the potential flaws in the literature. Furthermore, it demonstrates how the framework of $\mathbb{CLDFSS}$ generalizes all the existing theories and provides a wide range to deal with many real-world problems. Additionally, some beneficial operational laws were discussed alongside examples and theorems. A mathematical framework was designed for $\mathbb{CLDFSS}$ to select a pertinent antenna in a communication device to attain army sustainability goals. A comparative examination of existing theories with the proposed technique and their drawbacks was investigated. As a result, it is possible to conclude with confidence that the proposed method has improved stability and usability for decision-makers in the DM process based on the collected data.

In the future, the suggested $\mathbb{CLDFSS}$ technology is anticipated to address challenging problems in a variety of sectors. It can improve clustering and pattern recognition algorithms and produce more accurate results by efficiently handling ambiguous and partial data. In the medical industry, $\mathbb{CLDFSS}$ has the potential to enhance diagnostic accuracy through the integration of several data sources. Its application in economics and pattern recognition will enhance data analysis and predictions by identifying complex patterns within huge datasets. Furthermore, new theoretical tools for solving difficult issues will be made available through the development of advanced structures like topological, algebraic, and ordered structures within the $\mathbb{CLDFSS}$ framework.

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Conflicts of Interest

The authors declare no conflict of interest.

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